PARALLEL COPLANAR STRIPS
ON A DIELECTRIC SUBSTRATE

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THESIS

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ABSTRACT

A general analysis of coplanar lines on a dielectric substrate is presented. The field equations are written in terms of Hertzian potentials and boundary conditions are applied to generate a system of linear equations. These equations are then Fourier transformed and reduced to a form where a computationally efficient solution can be obtained by the method of moments. Any desired accuracy may be obtained by expanding the transform of the current if the problem is one involving strip conductors or the electric field if there are slots. A one term expansion produces very good results. Both wavelength and characteristic impedance of the transmission line structure are obtained. Theoretical and experimental results are presented for coplanar strips.

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I. INTRODUCTION

During the past several years a variety of coplanar microwave transmission lines has been developed covering a wide spectrum of applications. However, there still remained several possible structures to analyze, such as, coupled-slot lines, coplanar strips, etc. Also, theoretical analytic methods more accurate were sought to reduce the errors obtained by the more classical methods, such as the electrostatic approximation, the method of partial images, etc.

The present work is concerned with two parallel, coplanar strips over a dielectric substrate, the other side being bare, as illustrated in Fig. 1. The structure is uniform and infinite in both x and z directions; it is also assumed that the substrate material is lossless and its relative permittivity is ϵ_r.

For coplanar strips (CPS) to be practical as a transmission line, radiation must be minimized. This is accomplished through the use of a high permittivity substrate, which causes the coplanar strip wavelength to be small compared to the free-space wavelength, and thereby results in the fields being closely confined to the strips with negligible radiation loss.

A spectral domain transform method was suggested by Professors Itoh and Mittra [Refs. 1, 2] which yields an exact solution; this method together with the method of moments [Ref. 3], is used in this present work.

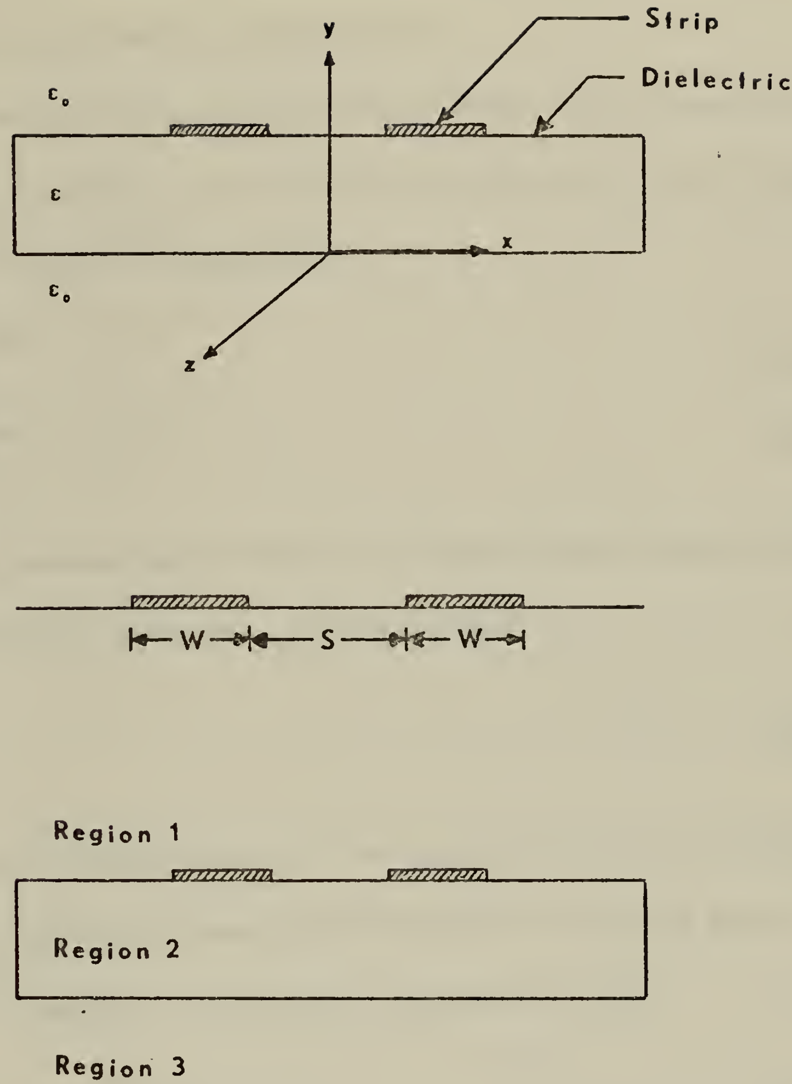


Fig. 1. Parallel coplanar strips configuration

II. DISPERSION CHARACTERISTICS

A. FIELD AND BOUNDARY CONDITIONS

Let the following electric and magnetic fields exist along the propagation or z-direction, as justified by the Hertzian vector potential functions described in Appendix A,

$$\mathcal{E}_z = k_c^2 \, \Phi^{(e)} e^{\gamma z} \tag{A1}$$

$$\mathcal{H}_z = k_c^2 \, \Phi^{(h)} e^{\gamma z} \tag{A2}$$

where γ is the propagation constant, and since it was stated that the substrate material was lossless, it follows that

$$\gamma = j\beta \quad . \tag{A3}$$

From the field expressions in equations (A1) and (A2), all other components of electric and magnetic fields can be derived from Maxwell's curl equations, as shown in Appendix B, as,

$$\mathcal{E}_x = \left(\gamma \frac{\partial \Phi^{(e)}}{\partial x} - j\omega\mu \frac{\partial \Phi^{(h)}}{\partial y} \right) e^{\gamma z} \tag{A4}$$

$$\mathcal{E}_y = \left(\gamma \frac{\partial \Phi^{(e)}}{\partial y} + j\omega\mu \frac{\partial \Phi^{(h)}}{\partial y} \right) e^{\gamma z} \tag{A5}$$

$$\mathcal{H}_x = \left(\gamma \frac{\partial \Phi^{(h)}}{\partial x} + j\omega\epsilon \frac{\partial \Phi^{(e)}}{\partial y} \right) e^{\gamma z} \tag{A6}$$

$$\mathcal{H}_y = \left(\gamma \frac{\partial \Phi^{(h)}}{\partial y} - j\omega\epsilon \frac{\partial \Phi^{(e)}}{\partial x} \right) e^{\gamma z} \tag{A7}$$

Applying boundary conditions at the interface between regions 2 and 3, tangential electric and magnetic fields must be continuous; therefore, at y = 0,

$$\mathcal{E}_{z_2}(x,0,z) = \mathcal{E}_{z_3}(x,0,z) \tag{A8}$$

$$\mathcal{E}_{x_2}(x,0,z) = \mathcal{E}_{x_3}(x,0,z) \tag{A9}$$

$$\mathcal{H}_{z_2}(x,0,z) = \mathcal{H}_{z_3}(x,0,z) \tag{A10}$$

$$\mathcal{H}_{x_2}(x,0,z) = \mathcal{H}_{x_3}(x,0,z) \quad . \tag{A11}$$

Similarly, at the interface between regions 1 and 2, tangential electric fields must be continuous and tangential magnetic fields discontinuous by corresponding surface current densities; therefore, at y = d,

$$\mathcal{E}_{z_1}(x,d,z) = \mathcal{E}_{z_2}(x,d,z) \tag{A12}$$

$$\mathcal{E}_{x_1}(x,d,z) = \mathcal{E}_{x_2}(x,d,z) \tag{A13}$$

$$\mathcal{H}_{z_1}(x,d,z) - \mathcal{H}_{z_2}(x,d,z) = \begin{cases} J_x(x)\, e^{\gamma z} & \text{on strips} \\ 0 & \text{elsewhere} \end{cases} \tag{A14}$$

$$\mathcal{H}_{x_1}(x,d,z) - \mathcal{H}_{x_2}(x,d,z) = \begin{cases} J_z(x)\, e^{\gamma z} & \text{on strips} \\ 0 & \text{elsewhere} \end{cases} \quad . \tag{A15}$$

Also, the electric fields will exist only in the dielectric part of the interface and can be expressed as,

$$\mathcal{E}_{z_1}(x,d,z) = \begin{cases} 0 & \text{on strips} \\ e_z(x)\, e^{\gamma z} & \text{elsewhere} \end{cases} \tag{A16}$$

$$\mathcal{E}_{x_1}(x,d,z) = \begin{cases} 0 & \text{on strips} \\ e_x(x)\, e^{\gamma z} & \text{elsewhere} \end{cases} \quad . \tag{A17}$$

Substituting the field expressions of equations (A1) through (A7) into the boundary conditions expressions of equations (A8) through (A17) one obtains,

$$k_{c_2}^2 \Phi_2^{(e)}(x,0) = k_{c_3}^2 \Phi_3^{(e)}(x,0) \tag{A18}$$

$$\gamma \frac{\partial \Phi_2^{(e)}}{\partial x}(x,0) - j\omega\mu_0 \frac{\partial \Phi_2^{(h)}}{\partial y}(x,0) = \gamma \frac{\partial \Phi_3^{(e)}}{\partial x}(x,0) - j\omega\mu_0 \frac{\partial \Phi_3^{(h)}}{\partial y}(x,0) \tag{A19}$$

$$k_{c_2}^2 \Phi_2^{(h)}(x,0) = k_{c_3}^2 \Phi_3^{(h)}(x,0) \tag{A20}$$

$$\gamma \frac{\partial \Phi_2^{(h)}}{\partial x}(x,0) + j\omega\epsilon_2 \frac{\partial \Phi_2^{(e)}}{\partial y}(x,0) = \gamma \frac{\partial \Phi_3^{(h)}}{\partial x}(x,0) + j\omega\epsilon_0 \frac{\partial \Phi_3^{(e)}}{\partial y}(x,0) \tag{A21}$$

$$k_{c_1}^2 \Phi_1^{(e)}(x,d) = k_{c_2}^2 \Phi_2^{(e)}(x,d) \tag{A22}$$

$$\gamma \frac{\partial \Phi_1^{(e)}}{\partial x}(x,d) - j\omega\mu_0 \frac{\partial \Phi_1^{(h)}}{\partial y}(x,d) = \gamma \frac{\partial \Phi_2^{(e)}}{\partial x}(x,d) - j\omega\mu_0 \frac{\partial \Phi_2^{(h)}}{\partial y}(x,d) \tag{A23}$$

$$k_{c_1}^2 \Phi_1^{(h)}(x,d) - k_{c_2}^2 \Phi_2^{(h)}(x,d) = J_x(x) \tag{A24}$$

$$\gamma \frac{\partial \Phi_1^{(h)}}{\partial x}(x,d) + j\omega\epsilon_0 \frac{\partial \Phi_1^{(e)}}{\partial y}(x,d) - \left[\gamma \frac{\partial \Phi_2^{(h)}}{\partial x}(x,d) + j\omega\epsilon_2 \frac{\partial \Phi_2^{(e)}}{\partial y}(x,d)\right] = J_z(x) \tag{A25}$$

$$k_{c_1}^2 \Phi_1^{(e)}(x,d) = e_z(x) \tag{A26}$$

$$\gamma \frac{\partial \Phi_1^{(e)}}{\partial x}(x,d) - j\omega\mu_0 \frac{\partial \Phi_1^{(h)}}{\partial y} = e_x(x) \quad . \tag{A27}$$

B. SPECTRAL DOMAIN TRANSFORM

All potential functions must satisfy the following relation:

$$\nabla_t^2 \Phi + k_c^2 \Phi = 0 \tag{B1}$$

where,

$$k_c^2 = \gamma^2 + k^2 = k^2 - \beta^2 . \tag{B2}$$

One introduces the Fourier transform to the α-domain, as suggested by Itoh and Mittra [Refs. 1, 2], via,

$$\tilde{\Phi}_i(\alpha, y) = \int_{-\infty}^{+\infty} \Phi_i(x, y)\, e^{j\alpha x}\, dx \quad , \quad i = 1,2,3 . \tag{B3}$$

Therefore, one can find the α-domain Fourier transform of equation (B1) as,

$$k_c^2\, \mathcal{F}_x\left[\Phi(x,y)\right] = -\mathcal{F}_x\left(\frac{\partial^2 \Phi}{\partial x^2}\right) - \mathcal{F}_x\left(\frac{\partial^2 \Phi}{\partial y^2}\right)$$

$$k_c^2\, \tilde{\Phi}(\alpha, y) = -(-j\alpha)^2\, \tilde{\Phi}(\alpha, y) - \frac{\partial^2 \tilde{\Phi}(\alpha, y)}{\partial y^2} \tag{B4}$$

$$\frac{\partial^2}{\partial y^2}\tilde{\Phi}(\alpha, y) = (\alpha^2 - k_c^2)\, \tilde{\Phi}(\alpha, y) . \tag{B5}$$

Furthermore, one can define propagation constants for each region,

$$\gamma_i^2 = \alpha^2 - k_{ci}^2 = \alpha^2 + \beta^2 - k_i^2 \tag{B6}$$

where,

$$k_1 = k_3 = \omega\sqrt{\mu_0 \epsilon_0}$$

$$k_2 = k_1 \sqrt{\epsilon_r} . \tag{B7}$$

From planar dielectric substrate theory, the effective permittivity is related to ϵ_o and ϵ_2 as,

$$\epsilon_o < \epsilon_{eff} < \epsilon_2 \quad \text{(B7a)}$$

Therefore, analyzing the expressions for propagation constants in equation (B6), for region 1 or 3,

$$\gamma_1^2 = \gamma_3^2 = \alpha^2 + \beta^2 - k_1^2 = \alpha^2 + \left(\frac{2\pi}{\lambda'}\right)^2 - \left(\frac{2\pi}{\lambda}\right)^2 \quad \text{(B8)}$$

where λ' is the dielectric's effective wavelength and is related to the free-space wavelength λ as,

$$\frac{\lambda}{\sqrt{\epsilon_r}} < \lambda' < \lambda \quad \text{(B8a)}$$

Replacing this last expression into equation (B8), one can find lower and upper bound values for γ_1^2 or γ_3^2 as,

$$\alpha^2 + (\epsilon_r - 1)\left(\frac{2\pi}{\lambda}\right)^2 \geq \gamma_1^2 \geq \alpha^2 \quad \text{(B8b)}$$

therefore, γ_1 and γ_3 are always real quantities, independent of the values of α.

Similarly, for region 2,

$$\gamma_2^2 = \alpha^2 + \beta^2 - k_2^2 = \alpha^2 + \left(\frac{2\pi}{\lambda'}\right)^2 - \left(\frac{2\pi}{\lambda/\sqrt{\epsilon_r}}\right)^2 \quad \text{(B9)}$$

where, as before, the upper and lower bound values for α are,

$$\alpha^2 \geq \gamma_2^2 \geq \alpha^2 + (1 - \epsilon_r)\left(\frac{2\pi}{\lambda}\right)^2 \quad \text{(B9a)}$$

It is clear that γ_2 will be imaginary for small values of α and real for large values of α. Specifically, γ_2 will be imaginary for

$$\alpha < \frac{2\pi}{\lambda}\sqrt{\epsilon_r - 1} \quad \text{(B9b)}$$

and real for

$$\alpha > \frac{2\pi}{\lambda}\sqrt{\epsilon_r - 1} \quad . \tag{B9c}$$

Solving the potential functions differential equation (B5) for each of the three regions one obtains, as derived in Appendix C, - Electric field potential functions:

$$\Phi_1^{(e)}(\alpha, y) = A^{(e)}(\alpha)\, e^{-\gamma_1 (y-d)} \tag{B10}$$

$$\Phi_2^{(e)}(\alpha, y) = B^{(e)}(\alpha) \sinh \gamma_2 y + C^{(e)}(\alpha) \cosh \gamma_2 y\ , \quad \gamma_2 = \mathcal{R}e \tag{B11}$$

$$\Phi_2^{(e)}(\alpha, y) = B_1^{(e)}(\alpha) \sin \gamma_2 y + C^{(e)}(\alpha) \cos \gamma_2 y\ , \quad \gamma_2 = \mathcal{I}m \tag{B11a}$$

$$\Phi_3^{(e)}(\alpha, y) = D^{(e)}(\alpha)\, e^{\gamma_3 y} \tag{B12}$$

- Magnetic field potential functions:

$$\Phi_1^{(h)}(\alpha, y) = A^{(h)}(\alpha)\, e^{-\gamma_1 (y-d)} \tag{B13}$$

$$\Phi_2^{(h)}(\alpha, y) = B^{(h)}(\alpha) \sinh \gamma_2 y + C^{(h)}(\alpha) \cosh \gamma_2 y\ , \quad \gamma_2 = \mathcal{R}e \tag{B14}$$

$$\Phi_2^{(h)}(\alpha, y) = B_1^{(h)}(\alpha) \sin \gamma_2 y + C^{(h)}(\alpha) \cos \gamma_2 y\ , \quad \gamma_2 = \mathcal{I}m \tag{B14a}$$

$$\Phi_3^{(h)}(\alpha, y) = D^{(h)}(\alpha)\, e^{\gamma_3 y} \tag{B15}$$

where, clearly:

$$\gamma_1 = \gamma_3 \tag{B16}$$

$$k_{c_1} = k_{c_3} \tag{B16a}$$

and,

$$B_1^{(h)}(\alpha) = j\, B^{(h)}(\alpha) \tag{B17}$$

$$B_1^{(e)}(\alpha) = j\, B^{(e)}(\alpha) \tag{B17a}$$

where both, $B^{(h)}_{(\alpha)}$ and $B^{(e)}_{(\alpha)}$, are evaluated at $\gamma_2 = j\,\gamma_2''$.

In all the following discussion, equations subscripted with lower case "a" will refer to γ_2 being imaginary.

The set of equations (B10) through (B15) can be interpreted physically as follows:

- Electric and magnetic fields are more concentrated in the dielectric substrate, region 2, and decay exponentially toward zero outwards into the surrounding media, regions 1 and 3.
- In the dielectric substrate itself, electric and magnetic fields and, hence, energy, are more concentrated in the lower values of α . Although there is no direct comparison of the domains α and χ , a set of plots of the potential functions indicate that energy is also confined to the vicinity of the strips.

An approximate graphical representation of the energy distribution within the dielectric substrate for a fixed value of dimension y is shown in Fig. 2.

It is convenient to obtain the Fourier transform of equations (A18) through (A26) as follows,

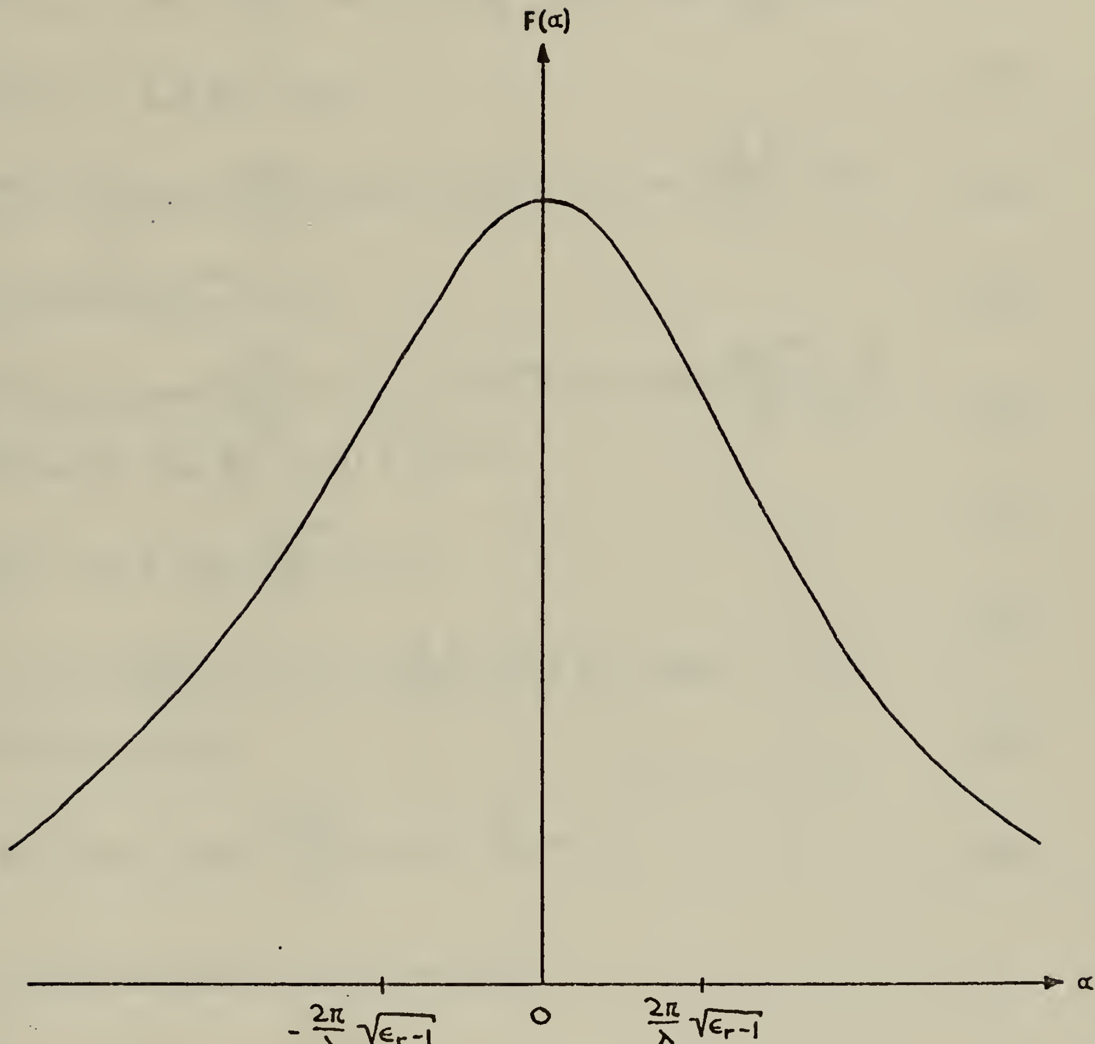


Fig. 2. Energy distribution within the dielectric substrate

$$k_{c_2}^2 \Phi_2^{(e)}(\alpha,0) = k_{c_3}^2 \Phi_3^{(e)}(\alpha,0) \tag{B18}$$

$$-j\alpha\gamma\Phi_2^{(e)}(\alpha,0) - j\omega\mu_0 \frac{\partial\Phi_2^{(h)}}{\partial y}(\alpha,0) = -j\alpha\gamma\Phi_3^{(e)}(\alpha,0) - j\omega\mu_0 \frac{\partial\Phi_3^{(h)}}{\partial y}(\alpha,0) \tag{B19}$$

$$k_{c_2}^2 \Phi_2^{(h)}(\alpha,0) = k_{c_3}^2 \Phi_3^{(h)}(\alpha,0) \tag{B20}$$

$$-j\alpha\gamma\Phi_2^{(h)}(\alpha,0) + j\omega\epsilon_2 \frac{\partial\Phi_2^{(e)}}{\partial y}(\alpha,0) = -j\alpha\gamma\Phi_3^{(h)}(\alpha,0) + j\omega\epsilon_0 \frac{\partial\Phi_3^{(e)}}{\partial y}(\alpha,0) \tag{B21}$$

$$k_{c_1}^2 \Phi_1^{(e)}(\alpha,d) = k_{c_2}^2 \Phi_2^{(e)}(\alpha,d) \tag{B22}$$

$$-j\alpha\gamma\Phi_1^{(e)}(\alpha,d) - j\omega\mu_0 \frac{\partial\Phi_1^{(h)}}{\partial y}(\alpha,d) = -j\alpha\gamma\Phi_2^{(e)}(\alpha,d) - j\omega\mu_0 \frac{\partial\Phi_2^{(h)}}{\partial y}(\alpha,d) \tag{B23}$$

$$k_{c_1}^2 \Phi_1^{(h)}(\alpha,d) - k_{c_2}^2 \Phi_2^{(h)}(\alpha,d) = J_x(\alpha) \tag{B24}$$

$$-j\alpha\gamma\Phi_1^{(h)}(\alpha,d) + j\omega\epsilon_0 \frac{\partial\Phi_1^{(e)}}{\partial y}(\alpha,d)$$

$$-\left[-j\alpha\gamma\Phi_2^{(h)}(\alpha,d) + j\omega\epsilon_2 \frac{\partial\Phi_2^{(e)}}{\partial y}(\alpha,d)\right] = J_z(\alpha) \tag{B25}$$

$$k_{c_1}^2 \Phi_1^{(e)}(\alpha,d) = \mathcal{E}_z(\alpha) \tag{B26}$$

$$-j\alpha\gamma\Phi_1^{(e)}(\alpha,d) - j\omega\mu_0 \frac{\partial\Phi_2^{(e)}}{\partial y}(\alpha,d) = \mathcal{E}_x(\alpha) \tag{B27}$$

where the derivative transform pair,

$$\mathcal{F}\ \frac{\partial\Phi}{\partial x} = -j\alpha\Phi \tag{B27a}$$

has been applied.

Upon substitution of the field expressions of equations (B10) through (B15) into the above relations, these become,

$$k_{c_2}^2\, C^{(e)}(\alpha) = k_{c_3}^2\, D^{(e)}(\alpha) \tag{B28}$$

$$\alpha\gamma\, C^{(e)}(\alpha) + \omega\mu_0\gamma_2\, B^{(h)}(\alpha) = \alpha\gamma\, D^{(e)}(\alpha) + \omega\mu_0\gamma_3\, D^{(h)}(\alpha) \tag{B29}$$

$$k_{c_2}^2\, C^{(h)}(\alpha) = k_{c_3}^2\, D^{(h)}(\alpha) \tag{B30}$$

$$\alpha\gamma\, C^{(h)}(\alpha) - \omega\epsilon_2\gamma_2\, B^{(e)}(\alpha) = \alpha\gamma\, D^{(h)}(\alpha) - \omega\epsilon_0\gamma_3\, D^{(e)}(\alpha) \tag{B31}$$

$$k_{c_1}^2\, A^{(e)}(\alpha) = k_{c_2}^2\left[B^{(e)}(\alpha)\sinh\gamma_2 d + C^{(e)}(\alpha)\cosh\gamma_2 d\right] \tag{B32}$$

$$\begin{aligned}\alpha\gamma\, A^{(e)}(\alpha) - \omega\mu_0\gamma_1\, A^{(h)}(\alpha) &= \alpha\gamma\left[B^{(e)}(\alpha)\sinh\gamma_2 d + C^{(e)}(\alpha)\cosh\gamma_2 d\right] \\ &\quad + \omega\mu_0\gamma_2\left[B^{(h)}(\alpha)\cosh\gamma_2 d + C^{(h)}(\alpha)\sinh\gamma_2 d\right]\end{aligned} \tag{B33}$$

$$k_{c_1}^2\, A^{(h)}(\alpha) - k_{c_2}^2\left[B^{(h)}(\alpha)\sinh\gamma_2 d + C^{(h)}(\alpha)\cosh\gamma_2 d\right] = J_x(\alpha) \tag{B34}$$

$$\begin{aligned}-j\alpha\gamma\, A^{(h)}(\alpha) - j\omega\epsilon_0\gamma_1\, A^{(e)}(\alpha) + j\alpha\gamma\left[B^{(h)}(\alpha)\sinh\gamma_2 d + C^{(h)}(\alpha)\cosh\gamma_2 d\right] \\ - j\omega\epsilon_2\gamma_2\left[B^{(e)}(\alpha)\cosh\gamma_2 d + C^{(e)}(\alpha)\sinh\gamma_2 d\right] = J_z(\alpha)\end{aligned} \tag{B35}$$

$$k_{c_1}^2\, A^{(e)}(\alpha) = \mathcal{E}_z(\alpha) \tag{B36}$$

$$-j\alpha\gamma\, A^{(e)}(\alpha) + j\omega\mu_0\gamma_1\, A^{(h)}(\alpha) = \mathcal{E}_x(\alpha) \tag{B37}$$

Equations (B28) through (B31) can be expressed in terms of $C^{(e)}_{(\alpha)}$ and $C^{(h)}_{(\alpha)}$, obtaining:

$$D^{(e)}(\alpha) = \left(\frac{k_{c2}}{k_{c3}}\right)^2 C^{(e)}(\alpha) \tag{B38}$$

$$D^{(h)}(\alpha) = \left(\frac{k_{c2}}{k_{c3}}\right)^2 C^{(h)}(\alpha) \tag{B39}$$

$$B^{(e)}(\alpha) = \frac{1}{\omega\epsilon_2\gamma_2}\left\{\alpha\gamma\left[1-\left(\frac{k_{c2}}{k_{c3}}\right)^2\right] C^{(h)}(\alpha) + \omega\epsilon_0\gamma_3\left(\frac{k_{c2}}{k_{c3}}\right)^2 C^{(e)}(\alpha)\right\} \tag{B40}$$

$$B^{(e)}(\alpha) = \frac{1}{j\omega\epsilon_2\gamma_2}\left\{\alpha\gamma\left[1-\left(\frac{k_{c2}}{k_{c3}}\right)^2\right]C^{(h)}(\alpha) + \omega\epsilon_0\gamma_3\left(\frac{k_{c2}}{k_{c3}}\right)^2 C^{(e)}(\alpha)\right\} \quad \text{(B40a)}$$

$$B^{(h)}(\alpha) = -\frac{1}{\omega\mu_0\gamma_2}\left\{\alpha\gamma\left[\left(\frac{k_{c2}}{k_{c3}}\right)^2 - 1\right]C^{(e)}(\alpha) + \omega\mu_0\gamma_3\left(\frac{k_{c2}}{k_{c3}}\right)^2 C^{(h)}(\alpha)\right\} \quad \text{(B41)}$$

$$B^{(h)}(\alpha) = \frac{1}{j\omega\mu_0\gamma_2}\left\{\alpha\gamma\left[\left(\frac{k_{c2}}{k_{c3}}\right)^2 - 1\right]C^{(e)}(\alpha) + \omega\mu_0\gamma_3\left(\frac{k_{c2}}{k_{c3}}\right)^2 C^{(h)}(\alpha)\right\} \quad \text{(B41a)}$$

where, in equations (B40a) and (B41a),

$\gamma_2 = j\gamma_2''$.

Furthermore, one can relate the constants $C^{(e)}(\alpha)$ and $C^{(h)}(\alpha)$ in terms of $A^{(e)}(\alpha)$ and $A^{(h)}(\alpha)$ via equations (B32) and B33), obtaining,

$$C^{(e)}(\alpha) = P_1 A^{(e)}(\alpha) - P_2 A^{(h)}(\alpha) \quad \text{(B42)}$$

$$C^{(h)}(\alpha) = P_3 A^{(e)}(\alpha) - P_4 A^{(h)}(\alpha) \quad \text{(B43)}$$

A relation between the constants $A^{(e)}(\alpha)$ and $A^{(h)}(\alpha)$ in terms of the surface current densities $J_x(\alpha)$ and $J_z(\alpha)$ can be established via equations (B34) and (B35), obtaining,

$$A^{(e)}(\alpha) = \frac{1}{Q_2 Q_3 + Q_1 Q_4}\left[Q_1 J_z(\alpha) - Q_3 J_x(\alpha)\right] \quad \text{(B44)}$$

$$A^{(h)}(\alpha) = \frac{1}{Q_2 Q_3 + Q_1 Q_4}\left[Q_2 J_z(\alpha) + Q_4 J_x(\alpha)\right] \quad \text{(B45)}$$

Finally, applying equations (B36) and (B37), the surface current densities can be related to the electric fields $\mathcal{E}_x(\alpha)$ and $\mathcal{E}_z(\alpha)$, obtaining,

$$M_1(\alpha,\beta)\, J_z(\alpha) + M_2(\alpha,\beta)\, J_x(\alpha) = \mathcal{E}_z(\alpha) \tag{B46}$$

$$M_3(\alpha,\beta)\, J_z(\alpha) + M_4(\alpha,\beta)\, J_x(\alpha) = \mathcal{E}_x(\alpha) \tag{B47}$$

where all the intermediate steps are fully derived in Appendix D.

C. PHYSICAL PARAMETERS

Up to this point, the problem has been stated in a quite general form, i.e., there is no dependence on the actual physical configuration other than the boundary surfaces.

In general, the surface current densities, $J_x(\alpha)$ and $J_z(\alpha)$, can be expressed as a train of known basis functions, [Ref. 3], as,

$$J_x(\alpha) = \sum_{i=1}^{\infty} a_i\, j_{x_i}(\alpha) \tag{C1}$$

$$J_z(\alpha) = \sum_{i=1}^{\infty} b_i\, j_{z_i}(\alpha) \,. \tag{C2}$$

Substituting the above expressions into equations (B46) and (B47), one obtains,

$$M_1(\alpha,\beta) \sum_{i=1}^{\infty} b_i\, j_{z_i}(\alpha) + M_2(\alpha,\beta) \sum_{i=1}^{\infty} a_i\, j_{x_i}(\alpha) = \mathcal{E}_z(\alpha) \tag{C3}$$

$$M_3(\alpha,\beta) \sum_{i=1}^{\infty} b_i\, j_{z_i}(\alpha) + M_4(\alpha,\beta) \sum_{i=1}^{\infty} a_i\, j_{x_i}(\alpha) = \mathcal{E}_x(\alpha) \,. \tag{C4}$$

Since the two conductors are sufficiently narrow, one may assume that the surface current density in the x-direction is zero, which will modify equations (C3) and (C4) as,

$$M_1(\alpha,\beta)\sum_{i=1}^{\infty} b_i\, j_{zi}(\alpha) = \mathcal{E}_z(\alpha) \tag{C5}$$

$$M_3(\alpha,\beta)\sum_{i=1}^{\infty} b_i\, j_{zi}(\alpha) = \mathcal{E}_x(\alpha) \tag{C6}$$

Furthermore, although theoretically one should perform a multi-term approximation to the surface current density in the z-direction, in this case a one term approximation is sufficient since a good assumption of this surface current density is a square pulse, i.e.,

$$M_1(\alpha,\beta)\, b_1\, j_{z1}(\alpha) = \mathcal{E}_z(\alpha) \tag{C7}$$

$$M_3(\alpha,\beta)\, b_1\, j_{z1}(\alpha) = \mathcal{E}_x(\alpha) \tag{C8}$$

or, substituting equations (C1) and (C2) for the case where i = 1,

$$M_1(\alpha,\beta)\, J_z(\alpha) = \mathcal{E}_z(\alpha) \tag{C9}$$

$$M_3(\alpha,\beta)\, J_z(\alpha) = \mathcal{E}_x(\alpha) \tag{C10}$$

It is clear that one only needs to work with either one of the above equations; consider equation (C9), for the present case.

As mentioned before, a good approximation to the z-direction surface current density is a square pulse; therefore, it is necessary to obtain the α -domain Fourier transform of $J_z(\alpha)$, shown in Fig. 3, as,

$$J_z(\alpha) = \int_{-\frac{s}{2}-w}^{-\frac{s}{2}} e^{j\alpha x}\,dx - \int_{\frac{s}{2}}^{\frac{s}{2}+w} e^{j\alpha x}\,dx$$

$$= \frac{1}{j\alpha}\left[e^{j\alpha x}\Big|_{-\frac{s}{2}-w}^{-\frac{s}{2}} + e^{j\alpha x}\Big|_{\frac{s}{2}+w}^{\frac{s}{2}} \right] \tag{C11}$$

$$= \frac{1}{j\alpha}\left[e^{-j\alpha\frac{s}{2}} - e^{-j\alpha(\frac{s}{2}+w)} + e^{j\alpha\frac{s}{2}} - e^{j\alpha(\frac{s}{2}+w)} \right]$$

$$= \frac{1}{j\alpha}\left[2\cos\alpha\frac{s}{2} - 2\cos\alpha\left(\frac{s}{2}+w\right) \right]$$

$$= \frac{j4}{\alpha}\sin\frac{\alpha}{2}(s+w)\sin\frac{\alpha}{2}w \;.$$

Therefore, equation (C9) becomes,

$$M_1(\alpha,\beta)\frac{j4}{\alpha}\sin\frac{\alpha}{2}(s+w)\sin\frac{\alpha}{2}w = \mathcal{E}_z(\alpha) \;. \tag{C12}$$

In general, for a strip configuration, the electric fields are not easily defined; therefore, it is convenient to make equation (C12) independent of $\mathcal{E}_z(\alpha)$. This could be done by taking an inner product with a function orthogonal to $\mathcal{E}_z(\alpha)$. Applying this concept, equation (C12) becomes,

$$\langle M_1(\alpha,\beta)J_z(\alpha), W(\alpha)\rangle = \langle \mathcal{E}_z(\alpha), W(\alpha)\rangle \;. \tag{C13}$$

Define a suitable inner product, such as,

$$\langle V(\alpha), W(\alpha)\rangle = \int_{-\infty}^{+\infty} V(\alpha)\,W(\alpha)\,d\alpha \;. \tag{C14}$$

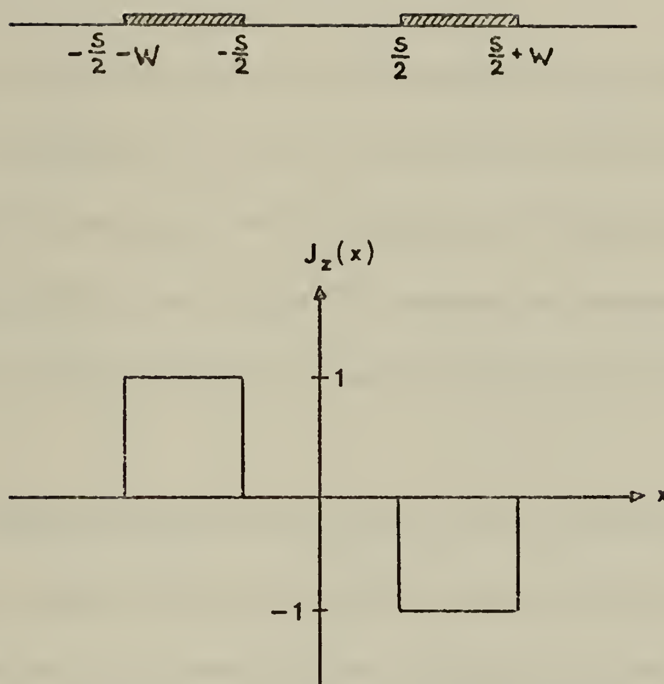


Fig. 3. Z - directed surface current density

A suitable weighting function $W(\alpha)$ could be the complex conjugate of $J_z(\alpha)$, i.e., $J_z(-\alpha)$.

Also, by Parseval's theorem, the right-hand side of equation (C13) becomes zero because of the orthogonality of the integrand.

Therefore, equation (C13) becomes,

$$16 \int_{-\infty}^{+\infty} M_1(\alpha,\beta) \frac{1}{\alpha^2} \sin^2 \frac{\alpha}{2} (s+w) \sin^2 \frac{\alpha}{2} w \, d\alpha = 0 . \tag{C15}$$

It is clear that once equation (C14) is integrated, the dependence on the variable α disappears, so one can state that the whole process is really a function of frequency and the structure's physical characteristics, namely, width of the strips W, separation between the strips S, thickness of the substrate D, and the dielectric's relative permittivity ϵ_r.

Once the α-domain Fourier transform of the surface current density is obtained, expressions for the different constants, A through D, have to be redefined to account for the imaginary character introduced by $J_z(\alpha)$. It turns out that all the magnetic potential constants, i.e., $A^{(h)}(\alpha)$, $B^{(h)}(\alpha)$, $C^{(h)}(\alpha)$, and $D^{(h)}(\alpha)$, are imaginary, and all the electric field constants, real. A detailed discussion of this procedure is presented in Appendix E.

Using the expressions derived in Appendix E, the equations for the various constants are modified as,

$$A^{(e)}(\alpha) = \frac{Q_1}{|Q_2| Q_3 - Q_1 |Q_4|} \left| J_z(\alpha) \right| \tag{C16}$$

$$A^{(h)}(\alpha) = j \frac{|Q_2|}{|Q_2|Q_3 - Q_1|Q_4|} |J_2(\alpha)| \tag{C17}$$

$$C^{(e)}(\alpha) = P_1 A^{(e)}(\alpha) + |P_2||A^{(h)}(\alpha)| \tag{C18}$$

$$C^{(h)}(\alpha) = j\left[|P_3|A^{(e)}(\alpha) - P_4|A^{(h)}(\alpha)|\right] \tag{C19}$$

$$B^{(e)}(\alpha) = \frac{1}{\omega\epsilon_2\gamma_2}\left\{\omega\epsilon_0\gamma_3\left(\frac{k_{c2}}{k_{c3}}\right)^2 C^{(e)}(\alpha) + \alpha\beta\left[\left(\frac{k_{c2}}{k_{c3}}\right)^2 - 1\right]|C^{(h)}(\alpha)|\right\} \tag{C20}$$

$$B^{(h)}(\alpha) = j\frac{1}{\omega\mu_0\gamma_2}\left\{\alpha\beta\left[\left(\frac{k_{c2}}{k_{c3}}\right)^2 - 1\right] C^{(e)}(\alpha) + \omega\mu_0\gamma_3\left(\frac{k_{c2}}{k_{c3}}\right)^2 |C^{(h)}(\alpha)|\right\} \tag{C21}$$

$$D^{(e)}(\alpha) = \left(\frac{k_{c2}}{k_{c3}}\right)^2 C^{(e)}(\alpha) \tag{C22}$$

$$D^{(h)}(\alpha) = j\left(\frac{k_{c2}}{k_{c3}}\right)^2 |C^{(h)}(\alpha)| . \tag{C23}$$

D. NUMERICAL INTEGRATION

Essentially, the main problem, as far as numerical integration is concerned, is the evaluation of equation (C15). Although theoretically the limits of integration are $-\infty$ and $+\infty$, truncation at points corresponding to sufficiently small values of the integrand is allowed.

The numerical integration method used is a modified Simpson's rule. Consider a general function, as shown in Fig. 4, where one divides the axis of integration α in N segments of width Δ ; the ordinate, corresponding to the half-width point of each segment, is taken as the average ordinate of each integration segment and a

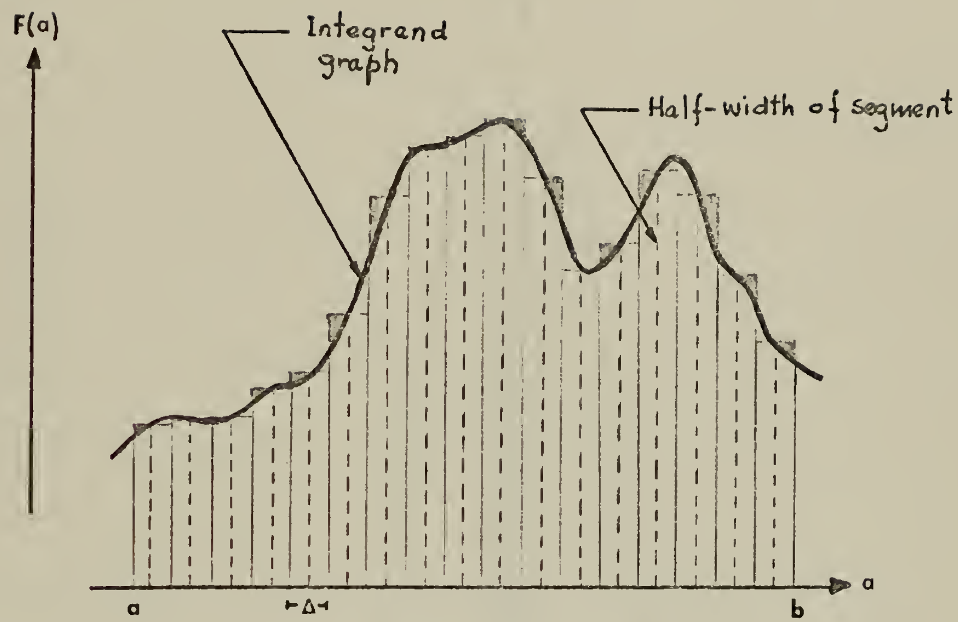


Fig. 4. Simpson's rule of numerical integration

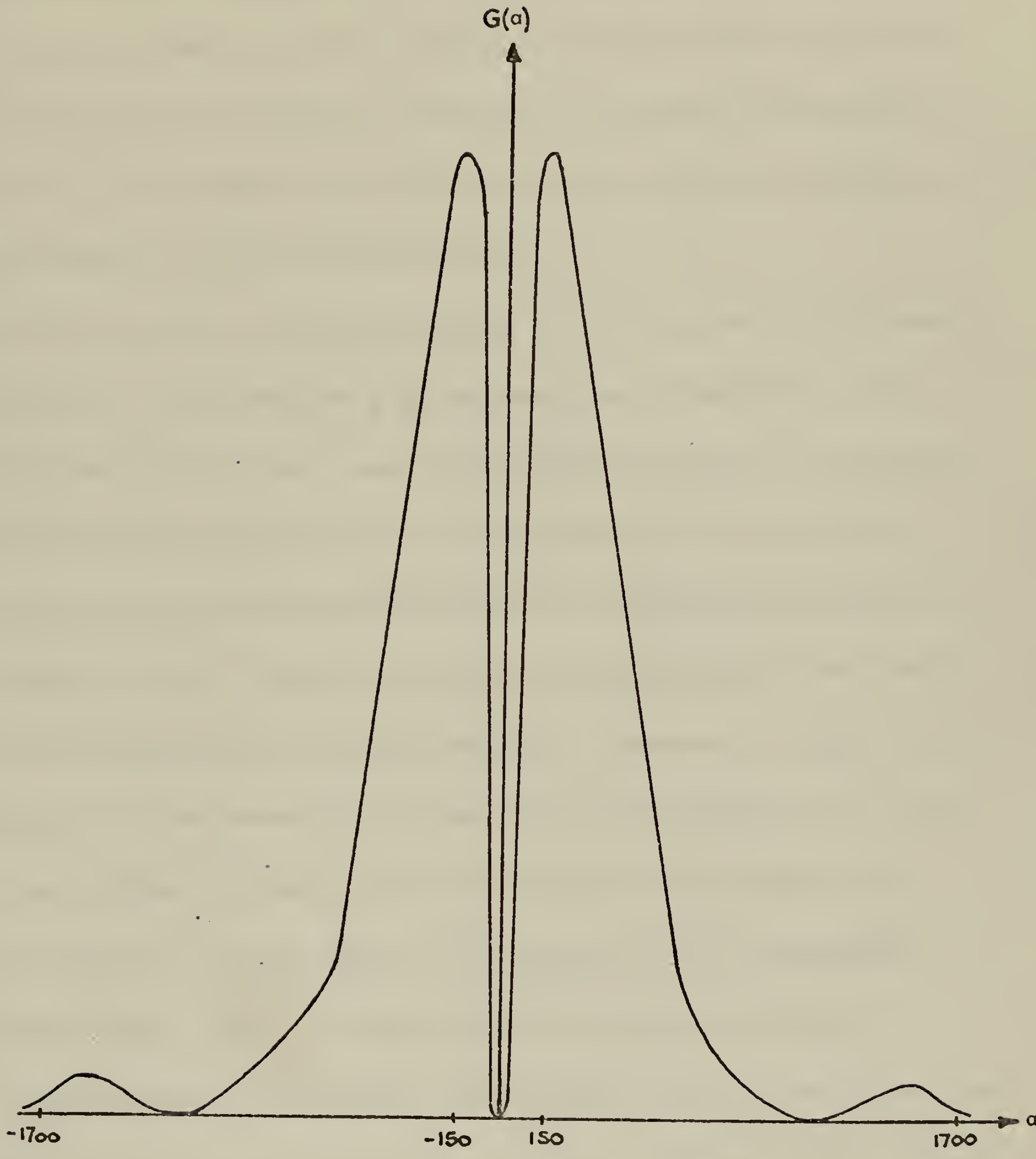


Fig. 5. Typical integrand plot

rectangle is formed with the segment's width. It follows that the total area under the given curve will be approximately equal to the sum of the areas comprised by the N rectangles.

It is clear that for more accurate results the width of each segment Δ should be a function of the rate of variation of the function $F(\alpha)$, i.e., for rapid-varying functions, the width Δ should be smaller than for slow-varying functions.

Several graphs of the integrand of the final equation (C15) were obtained; it is clear that for a given substrate of thickness D and relative permittivity ϵ_r, one can vary the frequency of operation, width of the strip conductors W, and separation between them S, obtaining different corresponding values of effective wavelengths on the dielectric λ'. Therefore, there is no single graph that could be used throughout the complete analysis. However, a graph corresponding to a set of parameters chosen arbitrarily will give a fairly good idea of how the integrand behaves and, consequently, will indicate the approximate limits of integration and an acceptable segment's width Δ. A typical graph is shown in Fig. 5.

It was chosen, according to the graphical expression of the integrand, that the limits of integration will be -1700 and +1700, since contributions beyond this point were less than 1% of the maximum contribution. Likewise, a good choice for the segment's width is 0.5, which will give a total of 6800 steps of integration.

The rate of change of the value of the integral in equation (C15) with respect to the effective dielectric wavelength λ' was also investigated to determine whether it was a monotonically increasing or decreasing function, an oscillating function, or a non-uniform function. For this purpose, the integral was evaluated at several increasing values of λ' and it was observed that it behaved as a monotonically decreasing function, as shown in Fig. 6.

The resulting integral in equation (C15),

$$\int_{-1700}^{+1700} M_1(\alpha, \beta) \frac{1}{\alpha^2} \sin^2 \frac{\alpha}{2}(s+w) \sin^2 \frac{\alpha}{2} W \, d\alpha$$

will reach the desired value of zero at an approximate value of for a given set of parameters, i. e., frequency, width of conductors, separation between conductors, thickness of the substrate, and dielectric permittivity.

Since β is defined as

$$\beta = \frac{2\pi}{\lambda'}$$

where λ' varies as

$$\frac{\lambda}{\sqrt{\epsilon_r}} < \lambda' < \lambda \ ,$$

it is clear that an iteration procedure should be used, varying from a starting value equal to λ_d, the dielectric's wavelength,

$$\lambda_d = \frac{\lambda}{\sqrt{\epsilon_r}}$$

in appropriate steps toward λ, until the value of zero is reached.

The accuracy of the effective dielectric's wavelength depends critically on the size of the above mentioned iteration steps.

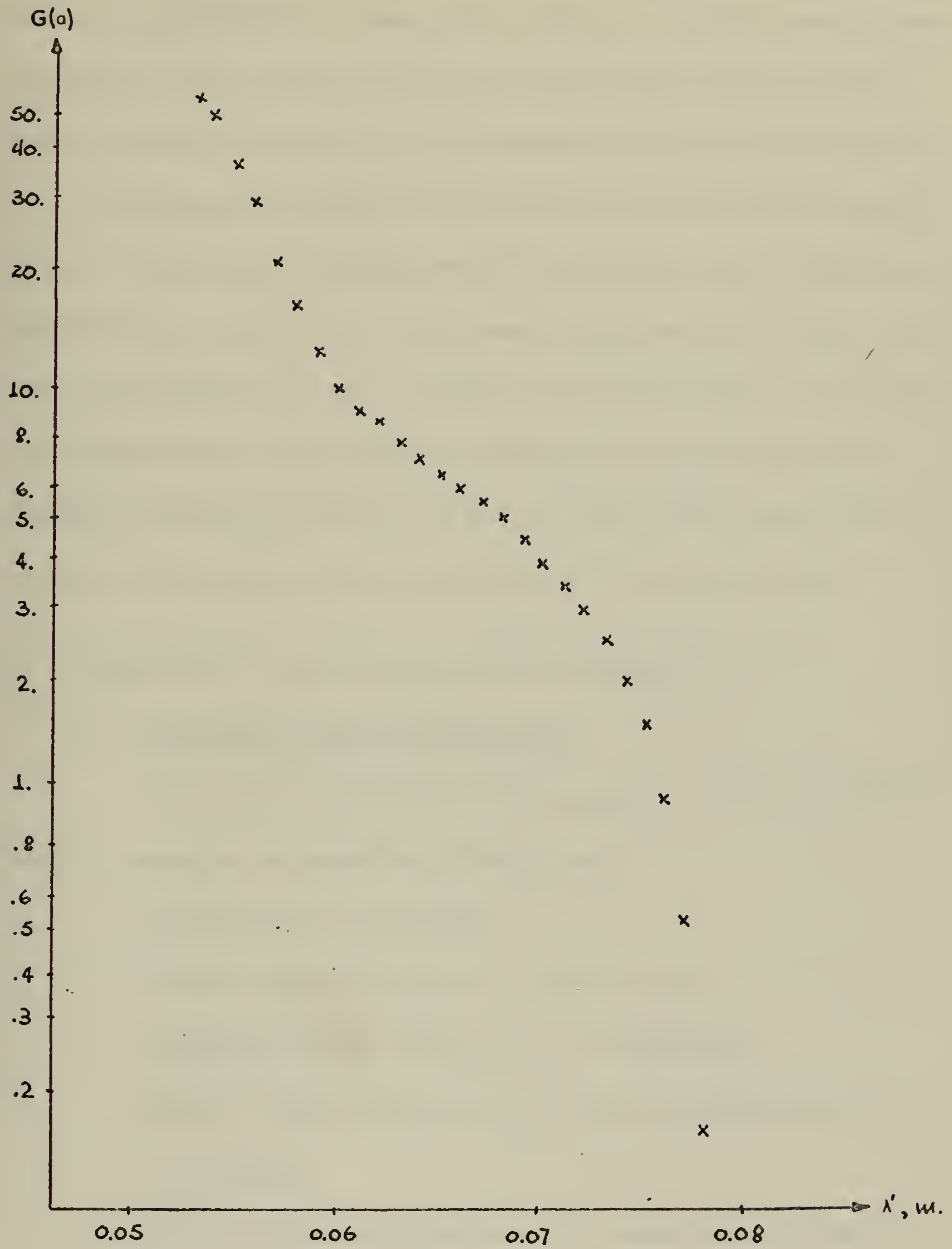


Fig. 6. Resulting integration variation with

Therefore, there is a trade-off in practice of accuracy vice computer time or, equivalently, accuracy vice dollars. For the present work accuracies of the order of 0.1% were obtained by using iteration steps equal to 1/1000 of the corresponding free-space wavelengths.

The integration subroutine itself was tried for several curves of known equations of various rates of variations, i.e., sinusoids, paraboloids, cubics, etc. In all cases variations of, at most, 0.2% their theoretical value were obtained. Also the limits of integration were increased to 2500 obtaining variations of, at most, 0.05%. Finally, integration steps of 0.4 and 0.6 were also tried to check accuracy and, again, variations of only 0.1% were observed.

E. COMPUTER PROGRAMMING AND RESULTS

1. Computer Program Organization

The dispersion characteristic program, written in FORTRAN IV language, accepts the following data:

- Dielectric's permittivity
- Width of the conductors, in millimeters
- Thickness of the substrate, in millimeters
- Ratio of the thickness of the substrate to free-space wavelength
- Ratio of the separation between conductors to width of conductors.

Likewise, the limits of integration, the step of integration , and the iteration step X, need to be specified.

The program starts by calculating all the required physical constants, namely, π , ϵ_0 , ϵ_2 , and μ_0 . All the α - and β-independent parameters are then evaluated, i. e., k_1, k_2, ω , etc. The iteration step is initiated by assigning the lower bound value of λ' , i. e., λ , defining in this way the value of β . At this point the integration is performed by assigning the lower bound value of α , i. e., -1700 for the present case defining consequently the parameters k_{c1}, k_{c2}, k_{c3}, γ_1, γ_2, and γ_3. Means are provided to test the real or imaginary character of γ_2 and two corresponding paths then follow. The integration, or better, the summation stops when α reaches its upper bound value, i. e., +1700 for the present case, and means are provided to test whether the absolute value of the resulting quantity is less than an arbitrary small epsilon value, i. e., 10^{-7} for the present case. If the case is that the result is greater than this epsilon value, λ' is increased toward λ in one iteration step X and the summation is performed again. The whole process ends either when a result less than epsilon is found, in which case the corresponding λ' is the answer sought, or when λ' reaches its upper bound value, i. e., λ .

The program is outlined in detail in Appendix F.

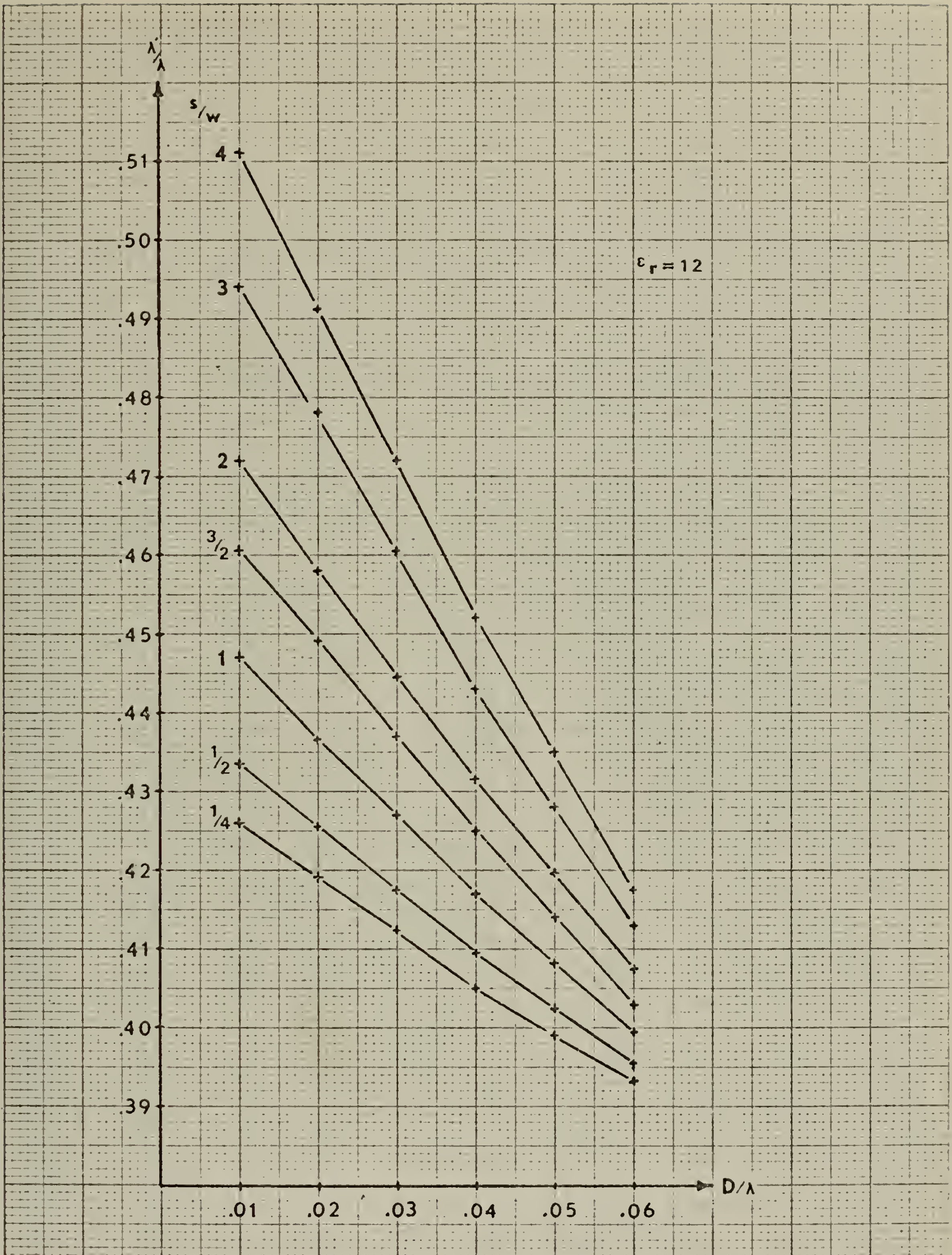


Fig. 7. Dispersion characteristic curves for $\epsilon_r = 12$

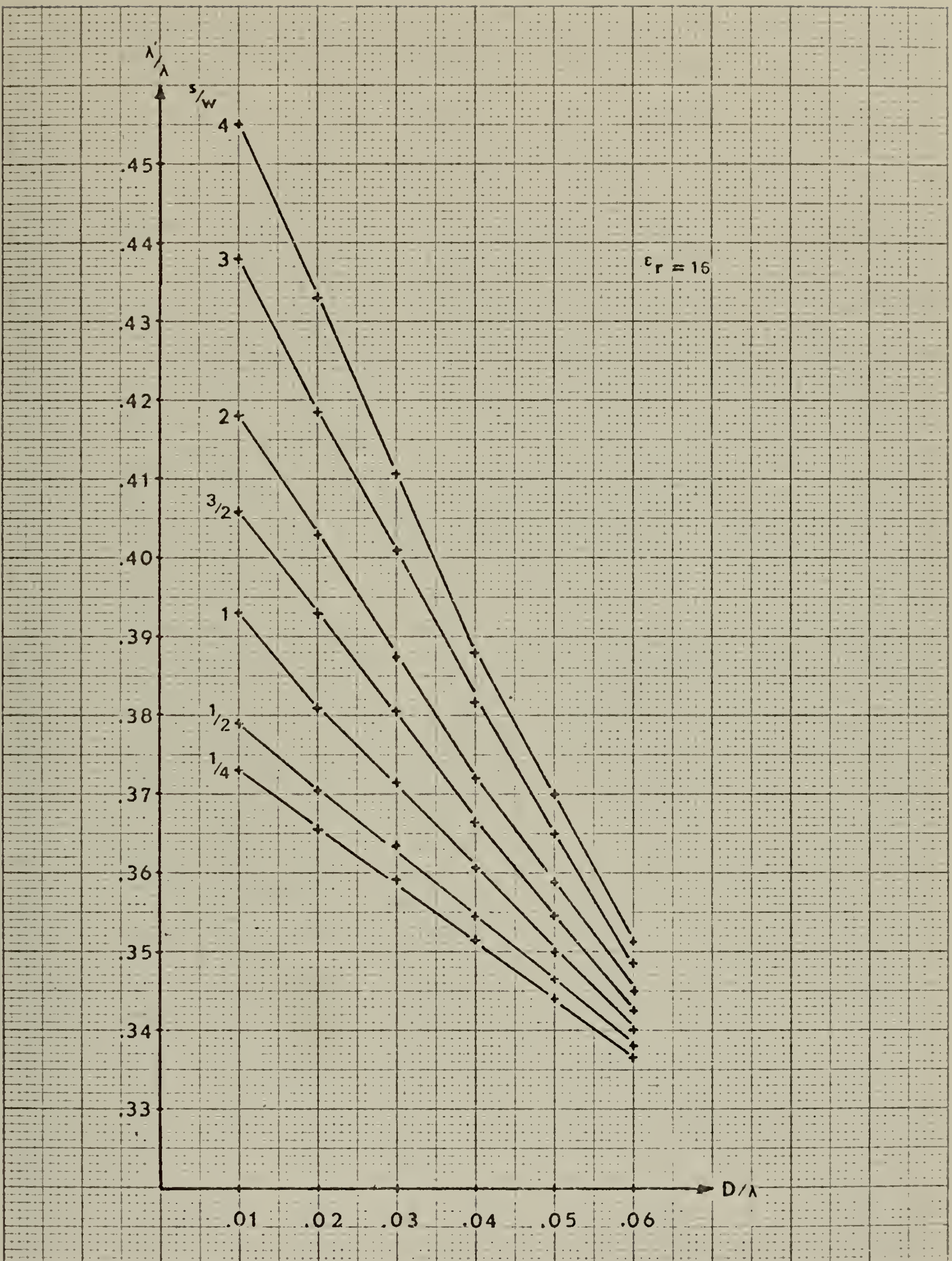


Fig. 8. Dispersion characteristic curves for $\epsilon_r = 16$

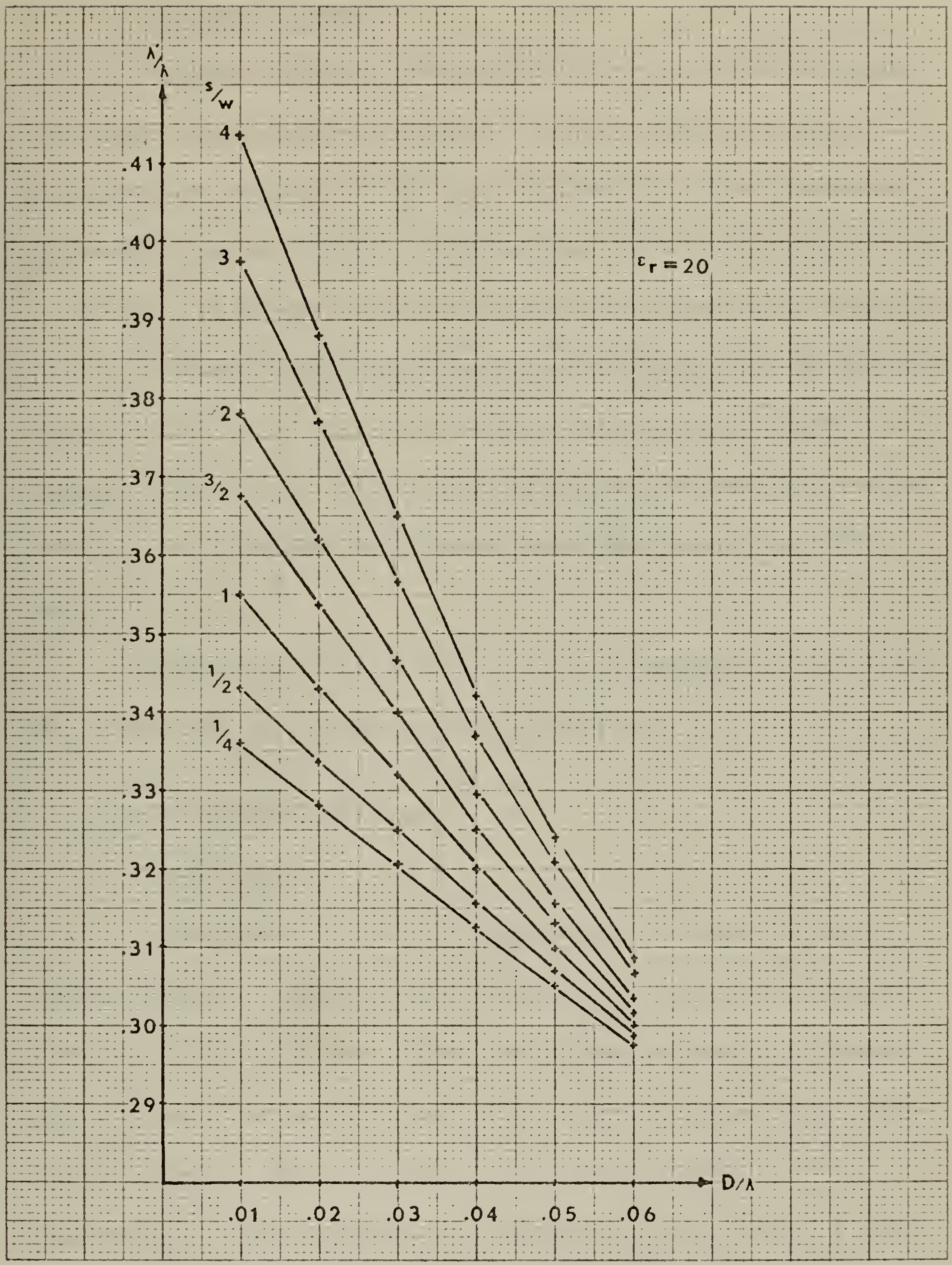


Fig. 9. Dispersion characteristic curves for $\epsilon_r = 20$

2. <u>Computer results</u>

Effective dielectric wavelength ratio relative to free-space wavelength information was obtained for three different relative permittivities, namely, for $\epsilon_r = 12$, 16, and 20. This information is presented in terms of ratios of the different parameters S, W, D, and in order to permit the use of any set of actual physical parameters.

For each permittivity, curves were obtained for the following values of the ratio of separation between conductors to width of conductors, $S/W = \frac{1}{4}$, $\frac{1}{2}$, 1, 3/2, 2, 3, and 4, corresponding to values of the ratio of substrate thickness to free-space wavelength D/λ from 0.01 to 0.06.

Dispersion curves for $\epsilon_r = 12$ are presented in Fig. 7, for $\epsilon_r = 16$ in Fig. 8, and for $\epsilon_r = 20$ in Fig. 9.

F. EXPERIMENTAL RESULTS

1. <u>Fabrication</u>

The coplanar strip (CPS) fabrication was made by photo-etching technique. Photoresist is first spread uniformly across the surface and allowed to dry. This film is then exposed to ultraviolet light through a photographic mask that permits the light to fall only on those areas in which the copper layer is preserved. The un-exposed film is next washed away from the surface, leaving a solid film of photoresist in the areas that were exposed. During the etching process the copper layer is removed in the unprotected areas

but is unaffected under the photoresist. Finally, the photoresist is removed with acetone, leaving only the dielectric surface with the two copper strips, as shown in Fig. 10.

It was decided to construct the coplanar strips on a piece of 6" x 3" x 1/8" copper clad substrate of relative permittivity of 12. Also, the ratio of width of conductors to thickness of the substrate was chosen arbitrarily as 1 and, similarly, the ratio of separation of conductors to width of conductors was also chosen arbitrarily as 1.

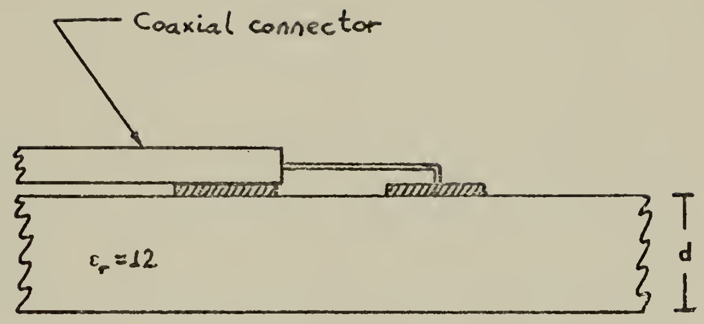
The coplanar strips were fed with a transition consisting of a 2½ inch piece of 85 mil semi-rigid coaxial line, soldered as shown in Fig. 10.

2. Experimental data

Basically, the measuring devices used were a slotted line fed by a sweep frequency oscillator and monitored by a voltage standing wave ratio meter.

From transmission line theory, it is known that an open - circuited line will have voltage nulls every half-wavelength, which in this case will be the effective dielectric wavelength.

Using this procedure, data was collected for a range of frequencies from 1 to 6 GHz and a comparison was made with the corresponding computer results, observing an error of 0.1% which was, as a matter of fact, the accuracy that was claimed for these results.



$d \approx 1/8''$

$\frac{S}{W} = 1$

$\frac{W}{d} = 1$

Fig. 10. Coplanar strips fabrication

Table I lists the experimental results and compares them with the computer results.

TABLE I

DISPERSION CHARACTERISTIC COMPARATIVE RESULTS

Coplanar strip of characteristics : $\epsilon_r = 12$

$d = 0.125''$

$s = w = d$

Frequency GHz	λ'/λ from Fig. 7	λ'/λ from experiment
1.0	0.447	0.446
2.0	0.435	0.435
3.0	0.425	0.424
4.0	0.415	0.414
5.0	0.406	0.406
6.0	0.398	0.399

G. THE AVERAGE POWER AS A FUNCTION OF THE HERTZIAN VECTOR POTENTIAL FUNCTIONS

As stated by Collin [Ref. 4], a general expression for the time-average power flow in terms of the electric and magnetic fields is

$$P_{AVE}(x,y) = \frac{1}{2} \mathrm{Re}\left[\iint_S \vec{\mathcal{E}} \times \vec{\mathcal{H}}^* \cdot \vec{a}_z \, ds \right] \tag{G1}$$

where,

$$\vec{\mathcal{E}} \times \vec{\mathcal{H}}^* \cdot \vec{a}_z = \mathcal{E}_x \mathcal{H}_y^* - \mathcal{E}_y \mathcal{H}_x^* \;. \tag{G2}$$

Referring back to equations (A1) through (A7), it is found that the equations for the electric and magnetic fields can be expressed in terms of Hertzian potential functions as,

$$\mathcal{E}_z = k_c^2 \, \Phi^{(e)} e^{\gamma z} \tag{G3}$$

$$\mathcal{H}_z = k_c^2 \, \Phi^{(h)} e^{\gamma z} \tag{G4}$$

$$\mathcal{E}_x = \left(\gamma \frac{\partial \Phi^{(e)}}{\partial x} - j\omega\mu_o \frac{\partial \Phi^{(h)}}{\partial y} \right) e^{\gamma z} \tag{G5}$$

$$\mathcal{E}_y = \left(\gamma \frac{\partial \Phi^{(e)}}{\partial y} + j\omega\mu_o \frac{\partial \Phi^{(h)}}{\partial x} \right) e^{\gamma z} \tag{G6}$$

$$\mathcal{H}_x = \left(\gamma \frac{\partial \Phi^{(h)}}{\partial x} + j\omega\epsilon \frac{\partial \Phi^{(e)}}{\partial y} \right) e^{\gamma z} \tag{G7}$$

$$\mathcal{H}_y = \left(\gamma \frac{\partial \Phi^{(h)}}{\partial y} - j\omega\epsilon \frac{\partial \Phi^{(e)}}{\partial x} \right) e^{\gamma z} \tag{G8}$$

So, substituting terms in the expression for average power,

$$P_{AVE}(x,y) = \frac{1}{2}\,\mathrm{Re}\Big[\iint_S \left(\mathcal{E}_x \mathcal{H}_y^* - \mathcal{E}_y \mathcal{H}_x^*\right) da\Big]$$

$$= \frac{1}{2}\,\mathrm{Re}\Big[\iint\Big\{\Big(j\beta \frac{\partial \Phi^{(e)}}{\partial x} - j\omega\mu_0 \frac{\partial \Phi^{(h)}}{\partial y}\Big)\Big(-j\beta \frac{\partial \Phi^{(h)*}}{\partial y} + j\omega\epsilon \frac{\partial \Phi^{(e)*}}{\partial x}\Big) \qquad \text{(G9)}$$

$$- \Big(j\beta \frac{\partial \Phi^{(e)}}{\partial y} + j\omega\mu_0 \frac{\partial \Phi^{(h)}}{\partial x}\Big)\Big(-j\beta \frac{\partial \Phi^{(h)*}}{\partial x} - j\omega\epsilon \frac{\partial \Phi^{(e)*}}{\partial y}\Big)\Big\}\, dx\, dy\Big]$$

Taking the α-domain Fourier transform of the average power expression,

$$P_{AVE}(\alpha,y) = \frac{1}{4\pi}\,\mathrm{Re}\Big[\iint\Big\{\Big(\alpha\beta\Phi^{(e)}(\alpha,y) - j\omega\mu_0 \frac{\partial \Phi^{(h)}}{\partial y}\Big)\Big(-j\beta \frac{\partial \Phi^{(h)*}}{\partial y} + \alpha\omega\epsilon\Phi^{(e)*}(\alpha,y)\Big) \qquad \text{(G10)}$$

$$- \Big(j\beta \frac{\partial \Phi^{(e)}}{\partial y} + \alpha\omega\mu_0 \Phi^{(h)}(\alpha,y)\Big)\Big(-\alpha\beta\Phi^{(h)*}(\alpha,y) - j\omega\epsilon \frac{\partial \Phi^{(e)*}}{\partial y}\Big)\Big\}\, d\alpha\, dy\Big]$$

where the Fourier transform of $\partial\Phi/\partial x$ has been applied, i. e.,

$$\mathcal{F}\left[\frac{\partial \Phi}{\partial x}\right] = -j\alpha\Phi \quad . \qquad \text{(G11)}$$

From equation (G10), performing the operations indicated,

$$P_{AVE}(\alpha,y) = \frac{1}{4\pi}\,\mathrm{Re}\Big[\iint_{-\infty}^{+\infty}\Big\{\alpha^2\beta\omega\epsilon\,\Phi^{(e)}(\alpha,y)\,\Phi^{(e)*}(\alpha,y) + \alpha^2\beta\omega\mu_0\,\Phi^{(h)}(\alpha,y)\,\Phi^{(h)*}(\alpha,y)$$

$$-\omega\epsilon\beta \frac{\partial \Phi^{(e)}}{\partial y}\frac{\partial \Phi^{(e)*}}{\partial y} - \omega\mu_0\beta \frac{\partial \Phi^{(h)}}{\partial y}\frac{\partial \Phi^{(h)*}}{\partial y} - j\alpha\beta^2\,\Phi^{(e)}(\alpha,y)\frac{\partial \Phi^{(h)*}}{\partial y} \qquad \text{(G12)}$$

$$-j\alpha k^2 \frac{\partial \Phi^{(h)}}{\partial y}\Phi^{(e)*}(\alpha,y) + j\alpha\beta^2 \frac{\partial \Phi^{(e)}}{\partial y}\Phi^{(h)*}(\alpha,y) + j\alpha k^2\,\Phi^{(h)}(\alpha,y)\frac{\partial \Phi^{(e)*}}{\partial y}\Big\}\, d\alpha\, dy\Big]$$

As far as the first four terms in the above expression for average power are concerned, they will be real. However, for the remaining four terms, since they correspond to cross-power terms, a more careful analysis has to be done. A detailed exposure to such analysis is given in Appendix G.

The real part of the average power turns out to be,

$$P_{AVE}(\alpha,y) = \frac{1}{4\pi}\iint_{-\infty}^{+\infty}\Big\{\omega\beta(\alpha^2-\gamma_1^2)\big(\epsilon_0\,\overline{A^{(e)}(\alpha)}^2+\mu_0|A^{(h)}(\alpha)|^2\big)e^{-2\gamma_1(y-d)}$$

$$+\omega\beta(\alpha^2-\gamma_3^2)\big(\epsilon_0\,\overline{D^{(e)}(\alpha)}^2+\mu_0|D^{(h)}(\alpha)|^2\big)e^{2\gamma_3 y}$$

$$+\sinh^2\gamma_2 y\Big[\alpha^2\beta\omega\big(\epsilon_2\,\overline{B^{(e)}(\alpha)}^2+\mu_0|B^{(h)}(\alpha)|^2\big)-\omega\beta\gamma_2^2\big(\epsilon_2\,\overline{C^{(e)}(\alpha)}^2+\mu_0|C^{(h)}(\alpha)|^2\big)$$

$$+\alpha\beta^2\gamma_2\big(|B^{(h)}(\alpha)|C^{(e)}(\alpha)-B^{(e)}(\alpha)|C^{(h)}(\alpha)|\big) \tag{G13}$$

$$+\alpha k_2^2\gamma_2\big(B^{(e)}(\alpha)|C^{(h)}(\alpha)|-|B^{(h)}(\alpha)|C^{(e)}(\alpha)\big)\Big]$$

$$+\cosh^2\gamma_2 y\Big[\alpha^2\beta\omega\big(\epsilon_2\,\overline{C^{(e)}(\alpha)}^2+\mu_0|C^{(h)}(\alpha)|^2\big)-\omega\beta\gamma_2^2\big(\epsilon_2\,\overline{B^{(e)}(\alpha)}^2+\mu_0|B^{(h)}(\alpha)|^2\big)$$

$$+\alpha\beta^2\gamma_2\big(B^{(e)}(\alpha)|C^{(h)}(\alpha)|-|B^{(h)}(\alpha)|C^{(e)}(\alpha)\big)$$

$$+\alpha k_2^2\gamma_2\big(|B^{(h)}(\alpha)|C^{(e)}(\alpha)-B^{(e)}(\alpha)|C^{(h)}(\alpha)|\big)\Big]$$

$$+\sinh 2\gamma_2 y\Big[\omega\beta\big(\epsilon_2 B^{(e)}(\alpha)C^{(e)}(\alpha)+\mu_0|B^{(h)}(\alpha)||C^{(h)}(\alpha)|\big)(\alpha^2-\gamma_2^2)\Big]\Big\}\,d\alpha\,dy$$

and similarly, for γ_2 imaginary,

$$P_{AVE}(\alpha,y) = \frac{1}{4\pi}\iint_{-\infty}^{+\infty}\Big\{\omega\beta(\alpha^2-\gamma_1^2)\big(\epsilon_0\,\overline{A^{(e)}(\alpha)}^2+\mu_0|A^{(h)}(\alpha)|^2\big)e^{-2\gamma_1(y-d)}$$

$$+\omega\beta(\alpha^2-\gamma_3^2)\big(\epsilon_0\,\overline{D^{(e)}(\alpha)}^2+\mu_0|D^{(h)}(\alpha)|^2\big)e^{2\gamma_3 y} \tag{G13a}$$

$$-\sin^2\gamma_2'' y\Big[\alpha^2\beta\omega\big(\epsilon_2\,\overline{B^{(e)}(\alpha)}^2+\mu_0|B^{(h)}(\alpha)|^2\big)+\omega\beta\gamma_2''^2\big(\epsilon_2\,\overline{C^{(e)}(\alpha)}^2+\mu_0|C^{(h)}(\alpha)|^2\big)$$

$$-\alpha\beta^2\gamma_2''\big(|B^{(h)}(\alpha)|C^{(e)}(\alpha)-B^{(e)}(\alpha)|C^{(h)}(\alpha)|\big)$$

$$-\alpha k_2^2\gamma_2''\big(B^{(e)}(\alpha)|C^{(h)}(\alpha)|-|B^{(h)}(\alpha)|C^{(e)}(\alpha)\big)\Big]$$

$$+\cos^2\gamma_2'' d\Big[\alpha^2\beta\omega\big(\epsilon_2\,\overline{C^{(e)}(\alpha)}^2+\mu_0|C^{(h)}(\alpha)|^2\big)+\omega\beta\gamma_2''^2\big(\epsilon_2\,\overline{B^{(e)}(\alpha)}^2+\mu_0|B^{(h)}(\alpha)|^2\big)$$

$$-\alpha\beta^2\gamma_2''\big(B^{(e)}(\alpha)|C^{(h)}(\alpha)|-|B^{(h)}(\alpha)|C^{(e)}(\alpha)\big)$$

$$-\alpha k_2^2\gamma_2''\big(|B^{(h)}(\alpha)|C^{(e)}(\alpha)-B^{(e)}(\alpha)|C^{(h)}(\alpha)|\big)\Big]\Big\}\,d\alpha\,dy$$

It is clear from the above expressions that one can integrate out the y-dependence of the average power, leaving only the α-dependence.

The integration details are shown in Appendix G, the resulting expressions being,

$$
\begin{aligned}
P_{AVE}(\alpha) = \frac{1}{8\pi}\int_{-\infty}^{+\infty} \Big\{ \omega\beta \Big[\Big(\frac{\alpha^2}{\gamma_1} - \gamma_1\Big)\Big[\epsilon_0 \Big(\overline{A^{(e)}(\alpha)}^{2} + \overline{D^{(e)}(\alpha)}^{2}\Big) \\
+ \mu_0 \Big(|A^{(h)}(\alpha)|^2 + |D^{(h)}(\alpha)|^2 \Big)\Big] \\
+ \frac{1}{2}\Big(\frac{\alpha^2}{\gamma_2} - \gamma_2\Big)\Big[\epsilon_2 \Big(\overline{B^{(e)}(\alpha)}^{2} + \overline{C^{(e)}(\alpha)}^{2}\Big) + \mu_0 \Big(|B^{(h)}(\alpha)|^2 + |C^{(h)}(\alpha)|^2\Big)\Big] \sinh 2\gamma_2 d \\
+ d(\alpha^2 + \gamma_2^2)\Big[\epsilon_2 \Big(\overline{C^{(e)}(\alpha)}^{2} - \overline{B^{(e)}(\alpha)}^{2}\Big) + \mu_0 \Big(|C^{(h)}(\alpha)|^2 - |B^{(h)}(\alpha)|^2\Big)\Big] \\
+ (\cosh 2\gamma_2 d - 1)\Big[\epsilon_2 B^{(e)}(\alpha) C^{(e)}(\alpha) + \mu_0 |B^{(h)}(\alpha)||C^{(h)}(\alpha)|\Big]\Big(\frac{\alpha^2}{\gamma_2} - \gamma_2\Big)\Big] \\
+ 2\alpha\gamma_2 \Big(|B^{(h)}(\alpha)| C^{(e)}(\alpha) - B^{(e)}(\alpha) |C^{(h)}(\alpha)|\Big)(k_2^2 - \beta^2)\, d \Big\}\, d\alpha
\end{aligned}
\tag{G14}
$$

and, similarly, for γ_2 imaginary,

$$
P_{AVE}(\alpha) = \frac{1}{8\pi}\int_{-\infty}^{+\infty} \Big\{ \omega\beta \Big[\Big(\frac{\alpha^2}{\gamma_1} - \gamma_1\Big)\Big[\epsilon_0 \Big(\overline{A^{(e)}(\alpha)}^{2} + \overline{D^{(e)}(\alpha)}^{2}\Big) \tag{G14a}
$$

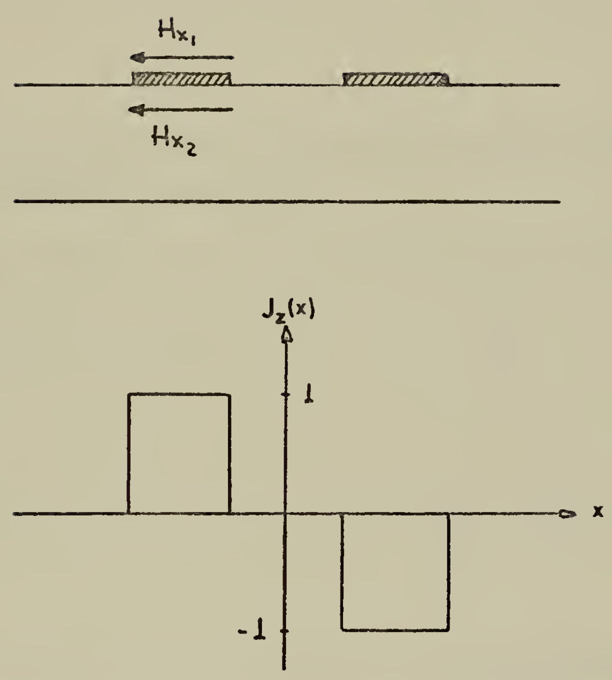


Fig. 11. Z - direction surface current density

$$+\mu_o\left(|A^{(h)}(\alpha)|^2+|D^{(h)}(\alpha)|^2\right)\Big]$$

$$+\frac{1}{2}\left(\frac{\alpha^2}{\gamma_2''}+\gamma_2''\right)\left[\epsilon_2\left(\overline{C^{(e)}(\alpha)}^2+\overline{B^{(e)}(\alpha)}^2\right)+\mu_o\left(|C^{(h)}(\alpha)|^2+|B^{(h)}(\alpha)|^2\right)\right]\sin 2\gamma_2'' d$$

$$+d\left(\alpha^2-\gamma_2''^2\right)\left[\epsilon_2\left(\overline{C^{(e)}(\alpha)}^2-\overline{B^{(e)}(\alpha)}^2\right)+\mu_o\left(|C^{(h)}(\alpha)|^2-|B^{(h)}(\alpha)|^2\right)\right]\Bigg]$$

$$-2\alpha\gamma_2''\left(B^{(e)}(\alpha)|C^{(h)}(\alpha)|-|B^{(h)}(\alpha)|C^{(e)}(\alpha)\right)\left(\beta^2+k_2^2\right)d\Big\}\,d\alpha$$

where in both equations (G14) and (G14a), the equality $\gamma_1 = \gamma_3$, has been applied.

H. THE CHARACTERISTIC IMPEDANCE IN TERMS OF THE DISPERSION CHARACTERISTICS

In general, one could express the average power in terms of the current in the coplanar strips as,

$$P_{AVE} = \frac{1}{2} I^2 Z_o \tag{H1}$$

from where,

$$Z_o = \frac{2P_{AVE}}{I^2} \,. \tag{H2}$$

In the present work, it was assumed in the dispersion characteristic part that the surface currents through the strips were uniform; furthermore, the current can be expressed, as shown in Fig. 11, as

$$\begin{aligned} I &= \oint \vec{H}\cdot d\vec{l} = \int_{strip} \left(H_{x_1} - H_{x_2}\right) dx \\ &= \int_{strip} |J_z(x)|\, dx \doteq W \end{aligned} \tag{H3}$$

where the approximation is valid since uniform surface current densities were assumed.

Furthermore, since it was also assumed a unit amplitude square pulse as the axial surface current density, i.e., $J_z(x) = 1$, it follows that,

$$I = W \tag{H4}$$

$$I^2 = W^2 . \tag{H4a}$$

All constants in the average power expressions of equations (G14) and (G14a) are completely defined by choosing appropriate values for the width of the strips, separation between strips, thickness of the slab, dielectric's permittivity, and frequency of operation.

The propagation constant β is now well-defined from the value of the corresponding effective dielectric's wavelength λ', i.e.,

$$\beta = \frac{2\pi}{\lambda'} \tag{H5}$$

where λ' is determined from the dispersion characteristic curves of Figs. 7, 8, and 9.

Therefore, the expressions to be evaluated for CPS characteristic impedance are,

$$\begin{aligned}
Z_o = \frac{1}{4\pi W^2} \int_{-\infty}^{+\infty} \Big\{ \omega\beta \Big[\Big(\frac{\alpha^2}{\gamma_1} - \gamma_1\Big) \Big[\epsilon_o \Big(\overline{A^{(e)}(\alpha)}^2 + \overline{D^{(e)}(\alpha)}^2\Big) \\
+ \mu_o \Big(|A^{(h)}(\alpha)|^2 + |D^{(h)}(\alpha)|^2\Big)\Big] \\
+ \frac{1}{2}\Big(\frac{\alpha^2}{\gamma_2} - \gamma_2\Big)\Big[\epsilon_2\Big(\overline{B^{(e)}(\alpha)}^2 + \overline{C^{(e)}(\alpha)}^2\Big) + \mu_o\Big(|B^{(h)}(\alpha)|^2 + |C^{(h)}(\alpha)|^2\Big)\Big] \sinh 2\gamma_2 d \\
+ d(\alpha^2 + \gamma_2^2)\Big[\epsilon_2\Big(\overline{C^{(e)}(\alpha)}^2 - \overline{B^{(e)}(\alpha)}^2\Big) + \mu_o\Big(|C^{(h)}(\alpha)|^2 - |B^{(h)}(\alpha)|^2\Big)\Big] \\
+ (\cosh 2\gamma_2 d - 1)\Big[\epsilon_2 B^{(e)}(\alpha) C^{(e)}(\alpha) + \mu_o |B^{(h)}(\alpha)||C^{(h)}(\alpha)|\Big]\Big(\frac{\alpha^2}{\gamma_2} - \gamma_2\Big)\Big] \\
+ 2\alpha\gamma_2 \Big(|B^{(h)}(\alpha)| C^{(e)}(\alpha) - B^{(e)}(\alpha)|C^{(h)}(\alpha)|\Big)(k_2^2 - \beta^2)\, d \Big\} \, d\alpha
\end{aligned} \tag{H6}$$

and, similarly, for the case where γ_2 is imaginary,

$$
\begin{aligned}
Z_0 = \frac{1}{4\pi W^2} \int_{-\infty}^{+\infty} \Big\{ \omega\beta \Big[\Big(\frac{\alpha^2}{\gamma_1} - \gamma_1 \Big) \Big[\epsilon_0 \big(A^{(e)}(\alpha) + D^{(e)}(\alpha) \big) \\
+ \mu_0 \big(|A^{(h)}(\alpha)|^2 + |D^{(h)}(\alpha)|^2 \big) \Big] \\
+ \frac{1}{2} \Big(\frac{\alpha^2}{\gamma_2''} + \gamma_2'' \Big) \Big[\epsilon_2 \big(C^{(e)}(\alpha) + B^{(e)}(\alpha) \big) + \mu_0 \big(|C^{(h)}(\alpha)|^2 + |B^{(h)}(\alpha)|^2 \big) \Big] \sin 2\gamma_2'' d \\
+ d \big(\alpha^2 - \gamma_2''^2 \big) \Big[\epsilon_2 \big(C^{(e)}(\alpha) - B^{(e)}(\alpha) \big) + \mu_0 \big(|C^{(h)}(\alpha)|^2 - |B^{(h)}(\alpha)|^2 \big) \Big] \Big] \\
- 2\alpha\gamma_2'' \big(B^{(e)}(\alpha) |C^{(h)}(\alpha)| - |B^{(h)}(\alpha)| C^{(e)}(\alpha) \big) \big(\beta^2 + k_2^2 \big) d \Big\} d\alpha
\end{aligned}
\qquad \text{(H6a)}
$$

I. NUMERICAL INTEGRATION

The numerical integration problem, as far as the characteristic impedance is concerned, is the evaluation of equations (H6) and (H6a). Again, although the limits of integration in α are $-\infty$ and $+\infty$, a prior knowledge of the behaviour of the integrands allows the use of truncated limits.

As in the dispersion characteristic part, the numerical integration method used is a modified Simpson's rule. Practical truncation limits are again -1700 and +1700 in steps of $\alpha = 0.5$, which will give 6800 computing points for a given set of parameters.

Once the physical parameters are specified, namely, the substrate's relative permittivity, the separation between conductors to width of the conductors ratio, and the substrate's thickness to free-space wavelength ratio, all the constants are well-defined except for the propagation constant β. However, this value can be interpolated

from the dispersion characteristic curves shown in part E. Therefore, numerically-wise, the calculation of the characteristic impedance is simpler than that for the dispersion characteristics since no iteration procedure is required.

J. COMPUTER PROGRAMMING AND RESULTS

1. Computer program organization

The characteristic impedance program, written in FORTRAN IV language, accepts the following data:

- Dielectric's relative permittivity
- Width of the conductors, in millimeters
- Thickness of the substrate, in millimeters
- Ratio of the thickness of the substrate to free-space wavelength
- Ratio of separation between conductors to width of conductors
- Ratio of effective dielectric's wavelength to free-space wavelength

Likewise, the limits of integration, and the step of integration need to be specified.

The program starts by calculating all constants and parameters and directly evaluates the impedance integral; basically, the procedure is the same as in the dispersion characteristic part except for the iteration search for λ' since now it is completely defined.

The program is outlined in detail in Appendix H.

2. Computer results

Characteristic impedance information was obtained for three different relative permittivities, namely, for ϵ_r = 12, 16, and 20.

As in the dispersion characteristic part, this information is presented in terms of ratios of the different parameters, i.e., S/W and D/λ, in order to permit the use of any set of actual physical parameters.

For each permittivity, curves were obtained for values of S/W equal to ¼, ½, 1, 3/2, 2, 3, and 4, corresponding to values of D/λ from 0.01 to 0.06.

Characteristic impedance curves for ϵ_r = 12 are presented in Fig. 12, for ϵ_r = 16 in Fig. 13, and for ϵ_r = 20 in Fig. 14.

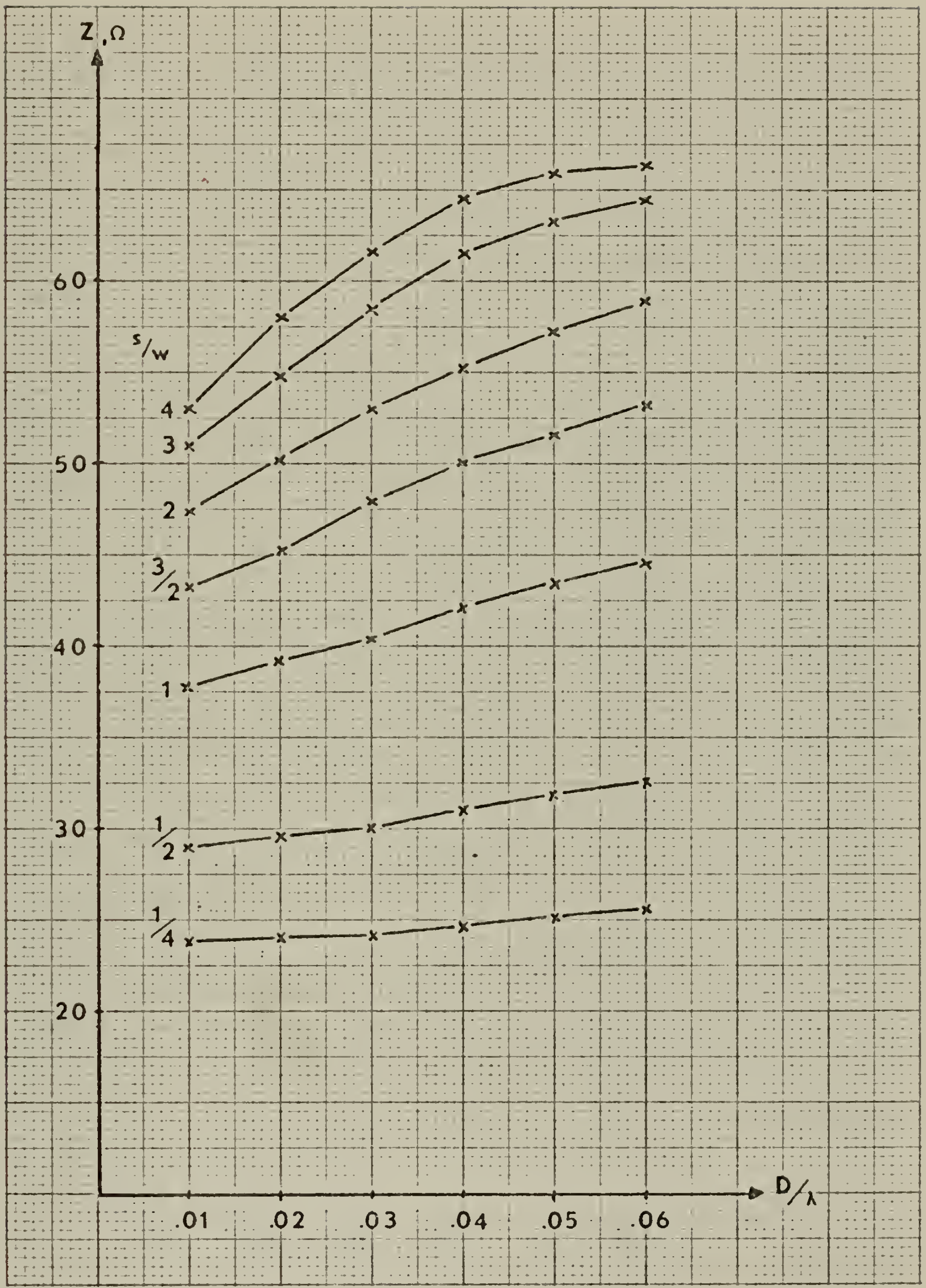


Fig. 12. Characteristic impedance curves for $\epsilon_r = 12$

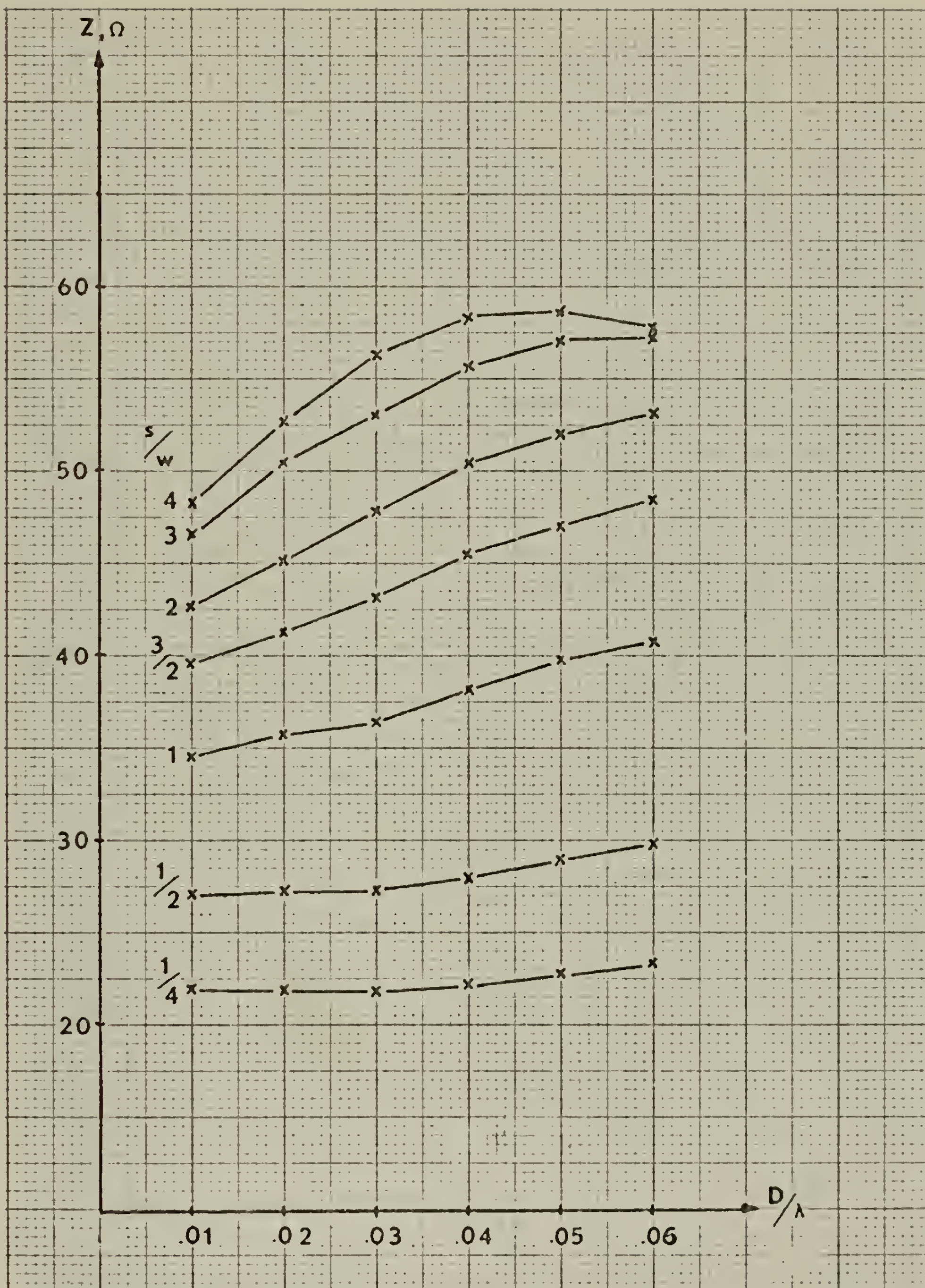


Fig. 13. Characteristic impedance curves for $\epsilon_r = 16$

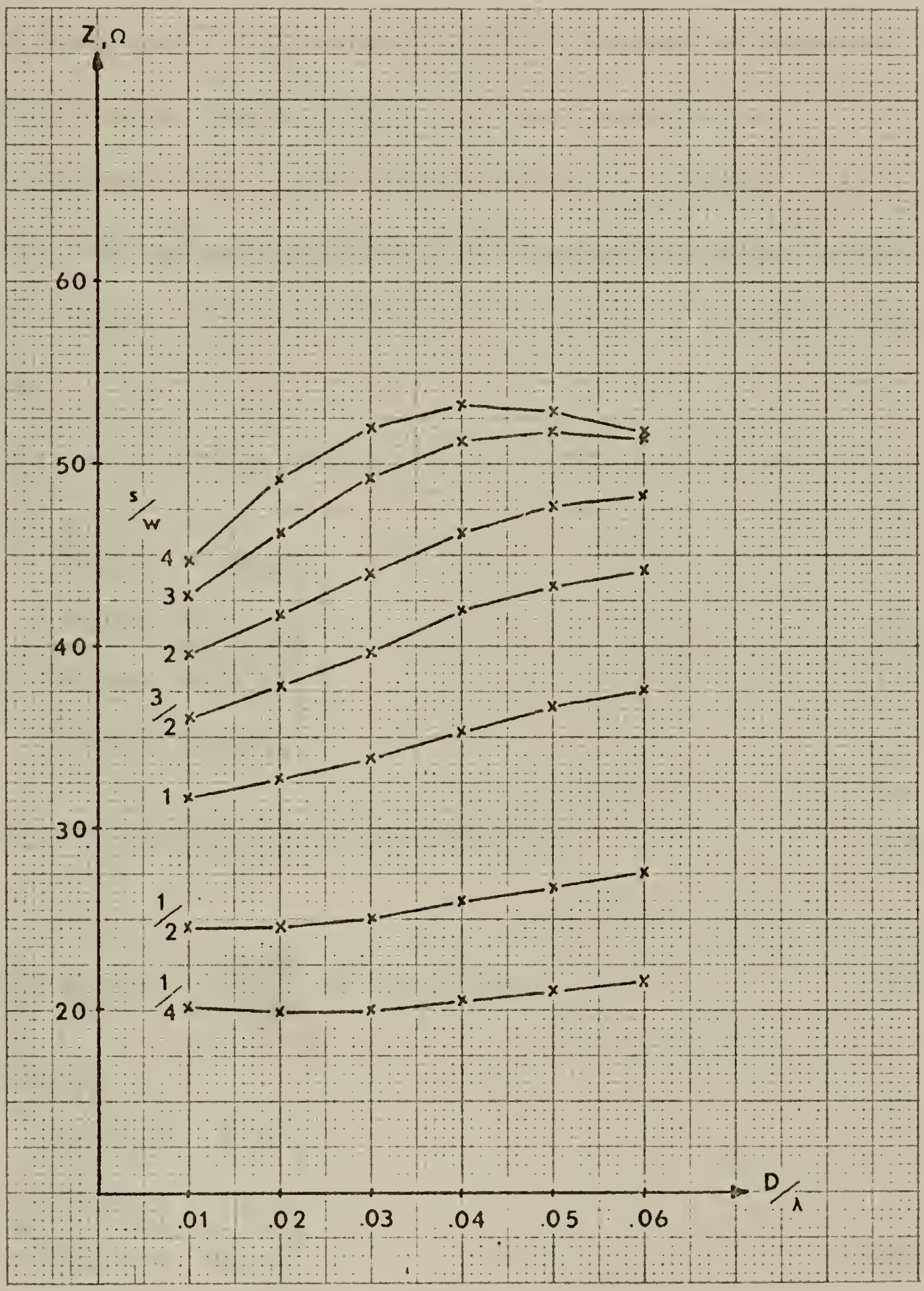


Fig. 14. Characteristic impedance curves for $\epsilon_r = 20$

IV. APPLICATIONS

The present method is applicable to problems based on any coplanar strip-type configuration and also to problems related to slot lines, such as coupled slots or coplanar waveguides.

For the latter case, the configuration is, in general, as shown in Fig. 15, and the main difference with the coplanar strip problem is the knowledge of the electric fields instead of the surface current densities.

Referring back to equations (B46) and (B47), these can be expressed as,

$$J_z(\alpha) = \frac{\begin{vmatrix} \mathcal{E}_z(\alpha) & M_2(\alpha,\beta) \\ \mathcal{E}_x(\alpha) & M_4(\alpha,\beta) \end{vmatrix}}{\begin{vmatrix} M_1(\alpha,\beta) & M_2(\alpha,\beta) \\ M_3(\alpha,\beta) & M_4(\alpha,\beta) \end{vmatrix}} \tag{L1}$$

where,

$$M_5(\alpha,\beta) = M_1(\alpha,\beta)\,M_4(\alpha,\beta) - M_2(\alpha,\beta)\,M_3(\alpha,\beta) \tag{L2}$$

therefore,

$$\frac{M_4(\alpha,\beta)}{M_5(\alpha,\beta)}\,\mathcal{E}_z(\alpha) - \frac{M_2(\alpha,\beta)}{M_5(\alpha,\beta)}\,\mathcal{E}_x(\alpha) = J_z(\alpha)\,. \tag{L3}$$

Similarly,

$$J_x(\alpha) = \frac{\begin{vmatrix} M_1(\alpha,\beta) & \mathcal{E}_z(\alpha) \\ M_3(\alpha,\beta) & \mathcal{E}_x(\alpha) \end{vmatrix}}{\begin{vmatrix} M_1(\alpha,\beta) & M_2(\alpha,\beta) \\ M_3(\alpha,\beta) & M_4(\alpha,\beta) \end{vmatrix}} \tag{L4}$$

and,

$$-\frac{M_3(\alpha,\beta)}{M_5(\alpha,\beta)}\,\mathcal{E}_z(\alpha)+\frac{M_1(\alpha,\beta)}{M_5(\alpha,\beta)}\,\mathcal{E}_x(\alpha)=J_x(\alpha) \quad . \tag{L5}$$

In general, for any slot-type configuration, z-directed electric fields can be neglected and the transverse, or x-directed electric fields can be approximated with square pulse-type distributions.

Unlike the coplanar-strip problem, here it is possible to obtain an odd and an even type of electric field excitations across the slots. In either case, equations (L3) and (L5) reduce to, respectively,

$$-\frac{M_2(\alpha,\beta)}{M_5(\alpha,\beta)}\,\mathcal{E}_{x_i}(\alpha)=J_z(\alpha) \tag{L6}$$

and

$$\frac{M_1(\alpha,\beta)}{M_5(\alpha,\beta)}\,\mathcal{E}_{x_i}(\alpha)=J_x(\alpha) \tag{L7}$$

where i = e, o depending on the type of excitation, even or odd, as shown in Fig. 16.

As far as the odd case of excitation is concerned, $\mathcal{E}_x(\alpha)$ assumes a form similar to $J_z(\alpha)$ in equation (C11), i.e.,

$$\mathcal{E}_{x_o}(\alpha)=\frac{j4}{\alpha}\sin\frac{\alpha}{2}(s+w)\sin\frac{\alpha}{2}w \quad . \tag{L8}$$

Performing the necessary Fourier transform and algebraic reductions becomes, for the even case of excitation,

$$\mathcal{E}_{x_e}(\alpha)=\frac{4}{\alpha}\cos\frac{\alpha}{2}(s+w)\sin\frac{\alpha}{2}w \, . \tag{L9}$$

Applying Galerkin's method and by Parseval's theorem, in the same approach as for the coplanar strip problem, the final iteration equation for the effective dielectric's wavelength becomes, considering only equation (L7),

$$\int_{-\infty}^{+\infty} \frac{M_1(\alpha,\beta)}{M_5(\alpha,\beta)} \mathcal{E}_{x_o}(\alpha)\, \mathcal{E}_{x_o}^{*}(\alpha)\, d\alpha = 0 \tag{L10}$$

or,

$$\int_{-\infty}^{+\infty} \frac{M_1(\alpha,\beta)}{M_5(\alpha,\beta)} \mathcal{E}_{x_e}(\alpha)\, \mathcal{E}_{x_e}^{*}(\alpha)\, d\alpha = 0 \tag{L11}$$

where $M_1(\alpha, \beta)$ through $M_4(\alpha, \beta)$ are defined in equations (122) through (125), respectively.

Remaining procedure is correspondingly similar to that for the coplanar strip case.

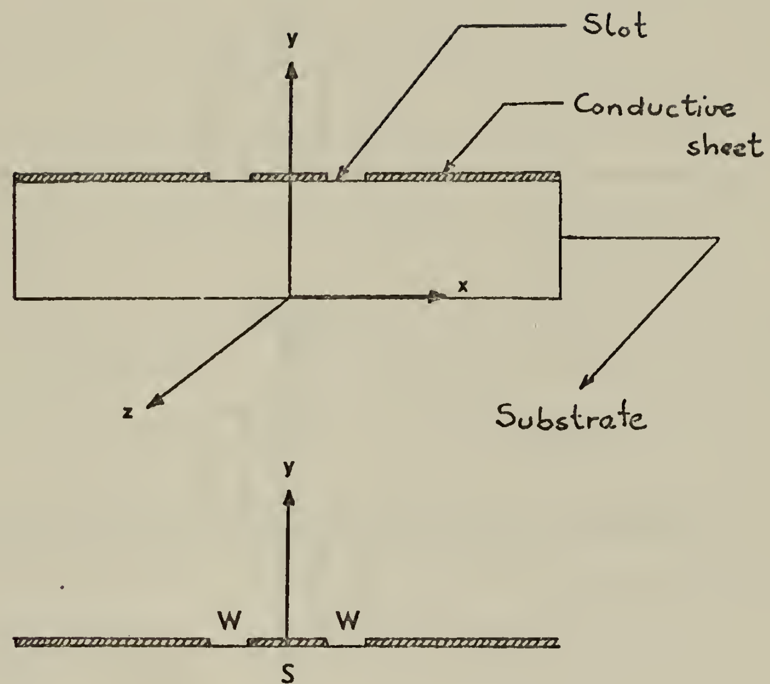


Fig. 15. Coupled slots configuration

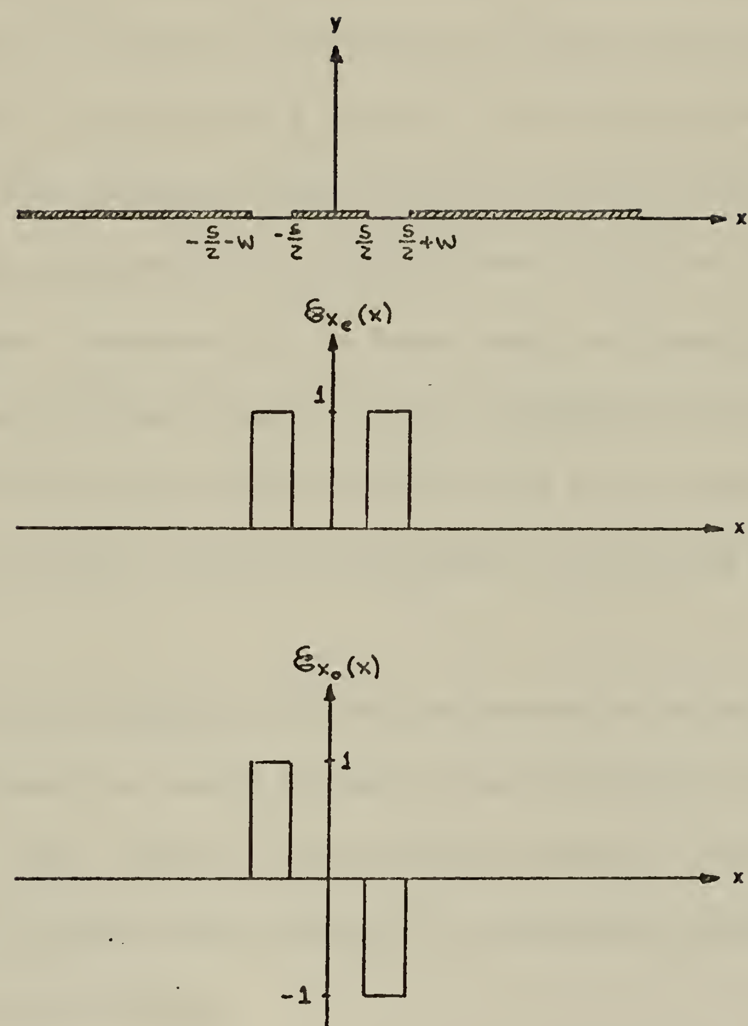


Fig. 16. Even and odd transverse electric fields

V. CONCLUSIONS

The method used in this present work succeeded in providing an accurate set of results for the dispersion characteristic and the characteristic impedance of a Coplanar Strip transmission line.

It was also discussed that an extension of this method will allow corresponding results for the Coupled Slots or Coplanar Waveguide configuration. However, for the latter case, care must be taken in adapting the equations to the even case of excitation due to the fact that the Fourier transform of the transverse electric field will now be real in character, instead of imaginary as in the odd case of excitation.

The amount of algebraic work can be reduced substantially by the use of matrices, which for the present configuration will have dimension eight. However, the algebraic approach used here gives much more insight to the problem and also reduces sharply the required computer time.

As far as the theoretical development is concerned, the only approximation used in the representation of the axial surface current density, i.e., a square pulse, and the value of zero assumed for the transverse surface current density. Depending on the desired accuracy, more complete, and indeed complex, representation of these surface current densities can be used.

In the numerical integration part of the problem, the integrating method itself has been proven to be sufficiently accurate. In fact, the only significant approximations used are the choice of the truncation limits, the integration step and the iteration step. However, several different sets of these choices were tried and, at least, for the present configuration, negligible variations were observed.

It is clear that the two programs used, i. e., the dispersion characteristic and the characteristic impedance, could be lumped together into one program reducing in about half the amount of computation and, therefore, the required computer time.

APPENDIX A

AUXILIARY VECTOR POTENTIAL FUNCTIONS

A. MAGNETIC HERTZIAN VECTOR POTENTIAL FUNCTION

Consider a homogeneous, source-free, isotropic region; hence, there is no charge density and

$$\vec{\nabla}\cdot\vec{\mathcal{E}} = 0 \quad . \tag{1}$$

From Vector Algebra, it is known that the divergence of a vector is zero if the vector is, in turn, the curl of another vector; therefore, one can state,

$$\vec{\nabla}\cdot\vec{\nabla}\times\vec{\pi}_h = 0 \qquad \forall\,\vec{\pi}_h \tag{2}$$

$$\vec{\mathcal{E}} = -j\omega\mu\vec{\nabla}\times\vec{\pi}_h \quad . \tag{3}$$

For time-varying fields

$$\vec{\nabla}\times\vec{\mathcal{H}} = j\omega\epsilon\,\vec{\mathcal{E}} \tag{4}$$

so applying equation (3),

$$\begin{aligned}\vec{\nabla}\times\vec{\mathcal{H}} &= j\omega\epsilon\,(-j\omega\mu\,\vec{\nabla}\times\vec{\pi}_h) \\ &= k^2\,\vec{\nabla}\times\vec{\pi}_h \\ &= k^2\,\vec{\nabla}\times(\vec{\pi}_h + k^{-2}\,\vec{\nabla}\phi)\end{aligned} \tag{5}$$

$$\vec{\mathcal{H}} = k^2\,\vec{\pi}_h + \vec{\nabla}\phi \tag{6}$$

where,

$$k^2 = \omega^2\mu\epsilon \; .$$

Similarly, for time-varying fields,

$$\vec{\nabla}\times\vec{\mathcal{E}} = -j\omega\mu\vec{\mathcal{H}} \tag{7}$$

so, applying equation (3),

$$\vec{\nabla}\times(-j\omega\mu\,\vec{\nabla}\times\vec{\pi}_h) = -j\omega\mu\,(k^2\,\vec{\pi}_h + \vec{\nabla}\phi) \tag{8}$$

$$\vec{\nabla}\vec{\nabla}\cdot\vec{\pi}_h - \nabla^2\vec{\pi}_h = k^2\,\vec{\pi}_h + \vec{\nabla}\phi \,. \tag{9}$$

Up to this point ϕ has not been defined; arbitrarily choose

$$\phi = \vec{\nabla}\cdot\vec{\pi}_h \,. \tag{10}$$

Then, equation (9) becomes,

$$\nabla^2\vec{\pi}_h + k^2\,\vec{\pi}_h = 0 \,. \tag{11}$$

Also, it is known that,

$$\begin{aligned} \vec{\nabla}\cdot\vec{B} &= \mu\,\vec{\nabla}\cdot\vec{\mathcal{H}} = 0 \\ \mu\,\vec{\nabla}\cdot(k^2\vec{\pi}_h + \vec{\nabla}\phi) &= 0 \\ k^2\,\vec{\nabla}\cdot\vec{\pi}_h + \vec{\nabla}\cdot\vec{\nabla}\phi &= 0 \end{aligned} \tag{12}$$

Summarizing, for TM modes,

$$\vec{\mathcal{E}} = -j\omega\mu\,\vec{\nabla}\times\vec{\pi}_h \tag{13}$$

$$\begin{aligned} \vec{\mathcal{H}} &= k^2\,\vec{\pi}_h + \vec{\nabla}\vec{\nabla}\cdot\vec{\pi}_h \\ &= \vec{\nabla}\times\vec{\nabla}\times\vec{\pi}_h \,. \end{aligned} \tag{14}$$

Similarly, for TE modes,

$$\vec{\mathcal{H}} = j\omega\epsilon\,\vec{\nabla}\times\vec{\pi}_h \tag{15}$$

$$\begin{aligned} \vec{\mathcal{E}} &= k^2\,\vec{\pi}_e + \vec{\nabla}\vec{\nabla}\cdot\vec{\pi}_e \\ &= \vec{\nabla}\times\vec{\nabla}\times\vec{\pi}_e \,. \end{aligned} \tag{16}$$

B. TE AND TM MODES FROM VECTOR POTENTIALS

1. TE Modes

$$\vec{\mathcal{E}} = -j\omega\mu\, \vec{\nabla}\times\vec{\pi}^h \tag{17}$$

$$\vec{\mathcal{H}} = k_c^2\, \vec{\pi}^h + \vec{\nabla}\vec{\nabla}\cdot\vec{\pi}^h \tag{18}$$

$$= \vec{\nabla}\times\vec{\nabla}\times\vec{\pi}^h$$

where $\vec{\pi}^h$ satisfies:

$$\nabla^2\vec{\pi}^h + k_c^2\,\vec{\pi}^h = 0 . \tag{19}$$

For TE modes, $\mathcal{E}_z = 0$; therefore,

$$\mathcal{E}_z = -j\omega\mu\left(\frac{\partial\pi_y^h}{\partial x} - \frac{\partial\pi_x^h}{\partial y}\right) \tag{20}$$

from where,

$$\pi_x^h = \pi_y^h = 0 . \tag{21}$$

Let:

$$\vec{\pi}^h = \phi^h e^{-\gamma z}\,\vec{a}_z . \tag{22}$$

Then, from equation (19),

$$\nabla_t^2\phi^h + k_c^2\,\phi^h = 0 \tag{23}$$

where,

$$k_c^2 = \gamma^2 + k^2 . \tag{24}$$

Summarizing,

$$\mathcal{H}_z = k_c^2\,\phi^h(x,y)\,e^{\pm\gamma z} \tag{25}$$

$$\vec{\mathcal{H}}_t = \pm\gamma e^{\pm\gamma z}\,\vec{\nabla}_t\,\phi^h \tag{26}$$

$$\vec{\mathcal{E}}_t = \pm\frac{j\omega\mu}{\gamma}\,\vec{a}_z\times\vec{\mathcal{H}}_t \tag{27}$$

2. TM Modes

$$\vec{\mathcal{E}} = k_c^2 \vec{\pi}^e + \vec{\nabla}\vec{\nabla}\cdot\vec{\pi}^e \tag{28}$$

$$= \vec{\nabla}\times\vec{\nabla}\times\vec{\pi}^e$$

$$\vec{\mathcal{H}} = j\omega\epsilon\, \vec{\nabla}\times\vec{\pi}^e \tag{29}$$

For TM Modes, $\mathcal{H}_z = 0$; therefore,

$$\vec{\pi}^e = \phi^e(x,y)\, e^{-\gamma z}\, \vec{a}_z \tag{30}$$

and,

$$\nabla_t^2 \phi^e + k_c^2 \phi^e = 0 \tag{31}$$

where,

$$k_c^2 = \gamma^2 + k^2 \tag{32}$$

Summarizing,

$$\mathcal{E}_z = k_c^2 \phi^e e^{\pm\gamma z} \tag{33}$$

$$\vec{\mathcal{E}}_t = \pm\gamma\vec{\nabla}_t \phi^e e^{\pm\gamma z} \tag{34}$$

$$\vec{\mathcal{H}}_t = \mp\frac{j\omega\epsilon}{\gamma}\,\vec{a}_z\times\vec{\mathcal{E}}_t \tag{35}$$

Equations (A1) and (A2) correspond to equations (33) and (25), respectively.

APPENDIX B

TRANSVERSE ELECTRIC AND MAGNETIC FIELDS

Consider Maxwell's point form of Ampere's law,

$$\vec{\nabla}\times\vec{\mathcal{H}} = j\omega\epsilon\vec{\mathcal{E}} \tag{36}$$

$$\vec{a}_x\left(\frac{\partial\mathcal{H}_z}{\partial y}-\frac{\partial\mathcal{H}_y}{\partial z}\right)+\vec{a}_y\left(\frac{\partial\mathcal{H}_x}{\partial z}-\frac{\partial\mathcal{H}_z}{\partial x}\right)+\vec{a}_z\left(\frac{\partial\mathcal{H}_y}{\partial x}-\frac{\partial\mathcal{H}_x}{\partial y}\right) = j\omega\epsilon\left(\mathcal{E}_x\vec{a}_x+\mathcal{E}_y\vec{a}_y+\mathcal{E}_z\vec{a}_z\right) \tag{37}$$

from where,

$$\frac{\partial\mathcal{H}_z}{\partial y}-\frac{\partial\mathcal{H}_y}{\partial z} = j\omega\epsilon\,\mathcal{E}_x \tag{38}$$

$$\frac{\partial\mathcal{H}_x}{\partial z}-\frac{\partial\mathcal{H}_z}{\partial x} = j\omega\epsilon\,\mathcal{E}_y \tag{39}$$

$$\frac{\partial\mathcal{H}_y}{\partial x}-\frac{\partial\mathcal{H}_x}{\partial y} = j\omega\epsilon\,\mathcal{E}_z \tag{40}$$

and, due to the z-dependence of the fields, i.e., $e^{\gamma z}$, one can apply,

$$\frac{\partial}{\partial z}e^{\gamma z} = \gamma e^{\gamma z} \tag{41}$$

obtaining from equations (38) through (40),

$$\frac{\partial\mathcal{H}_z}{\partial y}-\gamma\mathcal{H}_y = j\omega\epsilon\,\mathcal{E}_x \tag{42}$$

$$\gamma\mathcal{H}_x-\frac{\partial\mathcal{H}_z}{\partial x} = j\omega\epsilon\,\mathcal{E}_y \tag{43}$$

$$\frac{\partial\mathcal{H}_y}{\partial x}-\frac{\partial\mathcal{H}_x}{\partial y} = j\omega\epsilon\,\mathcal{E}_z\,. \tag{44}$$

Similarly, starting from Maxwell's point form of Lenz' law,

$$\vec{\nabla}\times\vec{\mathcal{E}} = -j\omega\mu\vec{\mathcal{H}} \tag{45}$$

$$\vec{a}_x\left(\frac{\partial\mathcal{E}_z}{\partial y}-\frac{\partial\mathcal{E}_y}{\partial z}\right)+\vec{a}_y\left(\frac{\partial\mathcal{E}_x}{\partial z}-\frac{\partial\mathcal{E}_z}{\partial x}\right)+\vec{a}_z\left(\frac{\partial\mathcal{E}_y}{\partial x}-\frac{\partial\mathcal{E}_x}{\partial y}\right)$$
$$= -j\omega\mu\left(\mathcal{H}_x\vec{a}_x+\mathcal{H}_y\vec{a}_y+\mathcal{H}_z\vec{a}_z\right) \tag{46}$$

from where,

$$\frac{\partial\mathcal{E}_z}{\partial y}-\gamma\,\mathcal{E}_y = -j\omega\mu\mathcal{H}_x \tag{47}$$

$$\gamma\,\mathcal{E}_x-\frac{\partial\mathcal{E}_z}{\partial x} = -j\omega\mu\mathcal{H}_y \tag{48}$$

$$\frac{\partial\mathcal{E}_y}{\partial x}-\frac{\partial\mathcal{E}_x}{\partial y} = -j\omega\mu\mathcal{H}_z\,. \tag{49}$$

Substituting the value of $\mathcal{E}_x$ from equation (48) into equation (42), one obtains,

$$\frac{\partial\mathcal{H}_z}{\partial y}-\gamma\mathcal{H}_y = \frac{j\omega\epsilon}{\gamma}\left(\frac{\partial\mathcal{E}_z}{\partial x}-j\omega\mu\,\mathcal{H}_y\right)$$

$$\mathcal{H}_y\left(\gamma^2+\omega^2\mu\epsilon\right) = \gamma\,\frac{\partial\mathcal{H}_z}{\partial y}-j\omega\epsilon\,\frac{\partial\mathcal{E}_z}{\partial x} \tag{50}$$

$$\mathcal{H}_y = \frac{1}{k_c^2}\left(\gamma\,\frac{\partial\mathcal{H}_z}{\partial y}-j\omega\epsilon\,\frac{\partial\mathcal{E}_z}{\partial x}\right).$$

Similarly, substituting the values of $\mathcal{E}_y$ from equation (47) into equation (43), of $\mathcal{H}_x$ from equation (43) into equation (47), and of $\mathcal{H}_y$ from equation (42) into equation (48), one obtains,

- For $\mathcal{H}_x$,

$$\gamma \mathcal{H}_x - \frac{\partial \mathcal{H}_z}{\partial x} = j\omega\epsilon \frac{1}{\gamma}\left(\frac{\partial \mathcal{E}_z}{\partial y} + j\omega\mu \mathcal{H}_x\right)$$

$$\mathcal{H}_x\left(\gamma^2+\omega^2\mu\epsilon\right) = \gamma \frac{\partial \mathcal{H}_z}{\partial x} + j\omega\epsilon \frac{\partial \mathcal{E}_z}{\partial y} \tag{51}$$

$$\mathcal{H}_x = \frac{1}{k_c^2}\left(\gamma \frac{\partial \mathcal{H}_z}{\partial x} + j\omega\epsilon \frac{\partial \mathcal{E}_z}{\partial y}\right).$$

- For $\mathcal{E}_y$,

$$\frac{\partial \mathcal{E}_z}{\partial y} - \gamma \mathcal{E}_y = -j\omega\mu \frac{1}{\gamma}\left(\frac{\partial \mathcal{H}_z}{\partial x} + j\omega\epsilon \mathcal{E}_y\right)$$

$$\mathcal{E}_y\left(\gamma^2+\omega^2\mu\epsilon\right) = \gamma \frac{\partial \mathcal{E}_z}{\partial y} + j\omega\mu \frac{\partial \mathcal{H}_z}{\partial x} \tag{52}$$

$$\mathcal{E}_y = \frac{1}{k_c^2}\left(\gamma \frac{\partial \mathcal{E}_z}{\partial y} + j\omega\mu \frac{\partial \mathcal{H}_z}{\partial x}\right).$$

- For $\mathcal{E}_x$,

$$\gamma^2 \mathcal{E}_x - \gamma \frac{\partial \mathcal{E}_z}{\partial x} = -j\omega\mu\left(\frac{\partial \mathcal{H}_z}{\partial y} - j\omega\epsilon \mathcal{E}_x\right)$$

$$\mathcal{E}_x\left(\gamma^2+\omega^2\mu\epsilon\right) = \gamma \frac{\partial \mathcal{E}_z}{\partial x} - j\omega\mu \frac{\partial \mathcal{H}_z}{\partial y} \tag{53}$$

$$\mathcal{E}_x = \frac{1}{k_c^2}\left(\gamma \frac{\partial \mathcal{E}_z}{\partial x} - j\omega\mu \frac{\partial \mathcal{H}_z}{\partial y}\right).$$

Summarizing,

$$\mathcal{E}_x = \frac{1}{k_c^2}\left(\gamma \frac{\partial \mathcal{E}_z}{\partial x} - j\omega\mu \frac{\partial \mathcal{H}_z}{\partial y}\right) \tag{54}$$

$$\text{(55)}\qquad \mathcal{E}_y = \frac{1}{k_c^2}\left(\gamma \frac{\partial \mathcal{E}_z}{\partial y} + j\omega\mu \frac{\partial \mathcal{H}_z}{\partial x}\right) \tag{55}$$

$$\text{(56)}\qquad \mathcal{H}_x = \frac{1}{k_c^2}\left(\gamma \frac{\partial \mathcal{H}_z}{\partial x} + j\omega\epsilon \frac{\partial \mathcal{E}_z}{\partial y}\right) \tag{56}$$

$$\text{(57)}\qquad \mathcal{H}_y = \frac{1}{k_c^2}\left(\gamma \frac{\partial \mathcal{H}_z}{\partial y} - j\omega\epsilon \frac{\partial \mathcal{E}_z}{\partial x}\right). \tag{57}$$

Equations (A4) through (A7) correspond to equations (54) through (57), respectively.

APPENDIX C

DERIVATION OF FIELD EQUATIONS

The differential equation defining the field functions is, as expressed in equation (B5),

$$\frac{\partial^2}{\partial y^2} \Phi_i(\alpha, y) - \gamma_i^2 \Phi_i(\alpha, y) = 0 \quad , \quad i = 1,2,3. \tag{58}$$

In turn, the propagation constant γ_i is expressed as,

$$\gamma_i^2 = \alpha^2 - k_{ci}^2 = \alpha^2 + \beta^2 - k_i^2 . \tag{59}$$

As stated before, the propagation constant for air medium, regions 1 and 3, is always real. However, for the dielectric medium, the propagation constant will be imaginary for small values of α and real for large values of α.

For region 1, the field function differential equation solution is as follows,

$$\frac{\partial^2}{\partial y^2} \Phi_1(\alpha, y) - \gamma_1^2 \Phi_1(\alpha, y) = 0$$

$$(s^2 - \gamma_1^2) \Phi_1(s) = 0 \tag{60}$$

$$\therefore \; \Phi_1(\alpha, y) = K_1 e^{+\gamma_1 y} + K_2 e^{-\gamma_1 y} .$$

Clearly, for infinite positive value of y, the field should vanish, or,

$$\lim_{y \to \infty} \Phi_1(\alpha, y) = 0$$

$$\therefore \; \Phi_1(\alpha, y) = K_2 e^{-\gamma_1 y} . \tag{61}$$

Similarly, for region 3,

$$\frac{\partial^2}{\partial y^2}\Phi_3(\alpha,y) - \gamma_3^2\,\Phi_3(\alpha,y) = 0 \qquad (62)$$

$$\Phi_3(\alpha,y) = K_3\,e^{+\gamma_3 y} + K_4\,e^{-\gamma_3 y}.$$

In this case, for infinite negative values of y, the field should vanish, or,

$$\lim_{y\to-\infty}\Phi_3(\alpha,y) = 0 \qquad (63)$$

$$\therefore\ \Phi_3(\alpha,y) = K_3\,e^{+\gamma_3 y}.$$

Finally, for region 2, there are two solutions, corresponding to the real or imaginary character of the propagation constant; the solution corresponding to the imaginary case will first be sought as follows,

$$\frac{\partial^2}{\partial y^2}\Phi_2(\alpha,y) + \gamma_2^2\,\Phi_2(\alpha,y) = 0$$

$$(s^2+\gamma_2^2)\,\Phi_2(s) = 0 \qquad (64)$$

$$s = \pm j\gamma_2$$

$$\therefore\ \Phi_2(\alpha,y) = K_5\sin\gamma_2 y + K_6\cos\gamma_2 y\ .$$

For values of α greater than $\frac{2\pi}{\lambda}\sqrt{\epsilon_r - 1}$, the propagation constant will be real, or

$$\Phi_2(\alpha,y) = K_7\sinh\gamma_2 y + K_8\cosh\gamma_2 y \qquad (65)$$

where,

$$K_7 = \pm j\,K_5 \qquad (66)$$

$$K_8 = K_6\ . \qquad (67)$$

One can define the following constants,

$$A(\alpha) = K_2$$
$$B(\alpha) = K_5$$
$$B_1(\alpha) = K_7$$
$$C(\alpha) = K_6 \qquad (68)$$
$$D(\alpha) = K_3$$

then, finally, applying the functions to the electric and magnetic field cases,

$$\Phi_1^{(e)}(\alpha,y) = A^{(e)}(\alpha)\, e^{-\gamma_1 y} \qquad (69)$$

$$\Phi_2^{(e)}(\alpha,y) = B^{(e)}(\alpha) \sinh \gamma_2 y + C^{(e)}(\alpha) \cosh \gamma_2 y \qquad (70)$$

$$\Phi_2^{(e)}(\alpha,y) = B_1^{(e)}(\alpha) \sin \gamma_2 y + C^{(e)}(\alpha) \cos \gamma_2 y \qquad (70a)$$

$$\Phi_3^{(e)}(\alpha,y) = D^{(e)}(\alpha)\, e^{\gamma_3 y} \qquad (71)$$

$$\Phi_1^{(h)}(\alpha,y) = A^{(h)}(\alpha)\, e^{-\gamma_1 y} \qquad (72)$$

$$\Phi_2^{(h)}(\alpha,y) = B^{(h)}(\alpha) \sinh \gamma_2 y + C^{(h)}(\alpha) \cosh \gamma_2 y \qquad (73)$$

$$\Phi_2^{(h)}(\alpha,y) = B_1^{(h)}(\alpha) \sin \gamma_2 y + C^{(h)}(\alpha) \cos \gamma_2 y \qquad (73a)$$

$$\Phi_3^{(h)}(\alpha,y) = D^{(h)}(\alpha)\, e^{\gamma_3 y} \qquad (74)$$

Equations (69) through (74) correspond to equations (B10) through (B15), respectively.

APPENDIX D

MATRIX CURRENT AND FIELD EQUATIONS DERIVATION

From equations (B32) and (B33), one could express the constants $A^{(e)}(\alpha)$ and $A^{(h)}(\alpha)$ in terms of $C^{(e)}(\alpha)$ and $C^{(h)}(\alpha)$ as,

$$A^{(e)}(\alpha) = \left(\frac{k_{c2}}{k_{c1}}\right)^2 \left\{ C^{(e)}(\alpha)\left[\frac{\epsilon_0\gamma_3}{\epsilon_2\gamma_2}\left(\frac{k_{c2}}{k_{c3}}\right)^2 \sinh\gamma_2 d + \cosh\gamma_2 d\right] - C^{(h)}(\alpha)\frac{\alpha\gamma}{\omega\epsilon_2\gamma_2}\left[\left(\frac{k_{c2}}{k_{c3}}\right)^2 - 1\right]\sinh\gamma_2 d \right\} \tag{75}$$

$$A^{(h)}(\alpha) = \frac{1}{\omega\mu_0\gamma_1}\left\{ C^{(e)}(\alpha)\,\alpha\gamma\left(\frac{k_{c2}}{k_{c3}}\right)^2 \frac{\epsilon_0\gamma_3}{\epsilon_2\gamma_2}\left[\left(\frac{k_{c2}}{k_{c3}}\right)^2 - 1\right]\sinh\gamma_2 d - C^{(h)}(\alpha)\left[\left((\alpha\gamma)^2\left[\left(\frac{k_{c2}}{k_{c3}}\right)^2 - 1\right]^2\frac{1}{\omega\epsilon_2\gamma_2} + \omega\mu_0\gamma_2\right)\sinh\gamma_2 d + \omega\mu_0\gamma_3\left(\frac{k_{c2}}{k_{c3}}\right)^2\cosh\gamma_2 d\right]\right\} . \tag{76}$$

Define the following constants:

$$N_1 = \left(\frac{k_{c2}}{k_{c3}}\right)^2\left[\frac{\epsilon_0\gamma_3}{\epsilon_2\gamma_2}\left(\frac{k_{c2}}{k_{c3}}\right)^2\sinh\gamma_2 d + \cosh\gamma_2 d\right] \tag{77}$$

$$N_2 = \left(\frac{k_{c2}}{k_{c3}}\right)^2\frac{\alpha\gamma}{\omega\epsilon_2\gamma_2}\left[\left(\frac{k_{c2}}{k_{c3}}\right)^2 - 1\right]\sinh\gamma_2 d \tag{78}$$

$$N_3 = \frac{\alpha\gamma}{\omega\mu_0\gamma_1}\left(\frac{k_{c2}}{k_{c3}}\right)^2\frac{\epsilon_0\gamma_3}{\epsilon_2\gamma_2}\left[\left(\frac{k_{c2}}{k_{c3}}\right)^2 - 1\right]\sinh\gamma_2 d \tag{79}$$

$$N_4 = \frac{1}{\omega\mu_0\gamma_1}\left\{\left[(\alpha\gamma)^2\left[\left(\frac{k_{c2}}{k_{c3}}\right)^2 - 1\right]\frac{1}{\omega\epsilon_2\gamma_2} + \omega\mu_0\gamma_2\right]\sinh\gamma_2 d + \omega\mu_0\gamma_3\left(\frac{k_{c2}}{k_{c3}}\right)^2\cosh\gamma_2 d\right\} . \tag{80}$$

Therefore, equations (75) and (76) become,

$$A^{(e)}(\alpha) = N_1\,C^{(e)}(\alpha) - N_2\,C^{(h)}(\alpha) \tag{81}$$

$$A^{(h)}(\alpha) = N_3\,C^{(e)}(\alpha) - N_4\,C^{(h)}(\alpha) \tag{82}$$

from where one could solve for $C^{(e)}(\alpha)$ and $C^{(h)}(\alpha)$ in terms of $A^{(e)}(\alpha)$ and $A^{(h)}(\alpha)$ as follows,

$$C^{(e)}(\alpha) = \frac{1}{N_1 N_4 - N_2 N_3}\left[N_4 A^{(e)}(\alpha) - N_2 A^{(h)}(\alpha)\right] \tag{83}$$

$$C^{(h)}(\alpha) = \frac{1}{N_1 N_4 - N_2 N_3}\left[N_3 A^{(e)}(\alpha) - N_1 A^{(h)}(\alpha)\right] . \tag{84}$$

Once again define the following constants:

$$P_1 = \frac{N_4}{N_1 N_4 - N_2 N_3} \tag{85}$$

$$P_2 = \frac{N_2}{N_1 N_4 - N_2 N_3} \tag{86}$$

$$P_3 = \frac{N_3}{N_1 N_4 - N_2 N_3} \tag{87}$$

$$P_4 = \frac{N_1}{N_1 N_4 - N_2 N_3} \tag{88}$$

Therefore, equations (83) and (84) can be expressed as,

$$C^{(e)}(\alpha) = P_1 A^{(e)}(\alpha) - P_2 A^{(h)}(\alpha) \tag{89}$$

$$C^{(h)}(\alpha) = P_3 A^{(e)}(\alpha) - P_4 A^{(h)}(\alpha) \tag{90}$$

which correspond to equations (B42) and (B43), respectively.

Substituting equations (89) and (90) into equations (B34) and (B35), one obtains,

$$\begin{aligned} A^{(h)}(\alpha) \Big\{ & k_{c2}^2 \left(\frac{\alpha \gamma}{\omega \mu_0 \gamma_2} \left[\left(\frac{k_{c2}}{k_{c3}} \right)^2 - 1 \right] P_2 + \frac{\gamma_3}{\gamma_2} \left(\frac{k_{c2}}{k_{c3}} \right)^2 P_4 \right) \sinh \gamma_2 d \\ & + k_{c2}^2 P_4 \cosh \gamma_2 d + k_{c1}^2 \Big\} \\ - k_{c2}^2 A^{(e)}(\alpha) \Big\{ & \left(\frac{\alpha \gamma}{\omega \mu_0 \gamma_2} \left[\left(\frac{k_{c2}}{k_{c3}} \right)^2 - 1 \right] P_1 + \frac{\gamma_3}{\gamma_2} \left(\frac{k_{c2}}{k_{c3}} \right)^2 P_3 \right) \sinh \gamma_2 d \\ & + P_3 \cosh \gamma_2 d \Big\} = J_x(\alpha) \end{aligned} \tag{91}$$

$$A^{(h)}(\alpha)\Big\{\sinh\gamma_2 d\Big[j\omega\epsilon_2\gamma_2 P_2 - j\frac{(\alpha\gamma)^2}{\omega\mu_0\gamma_2}\Big[\Big(\frac{k_{c2}}{k_{c3}}\Big)^2-1\Big]P_2$$

$$-j\alpha\gamma\frac{\gamma_3}{\gamma_2}\Big(\frac{k_{c2}}{k_{c3}}\Big)^2 P_4\Big] + \cosh\gamma_2 d\Big[j\omega\epsilon_0\gamma_3\Big(\frac{k_{c2}}{k_{c3}}\Big)^2 P_2 \tag{92}$$

$$-j\alpha\gamma P_4 - j\alpha\gamma\Big[\Big(\frac{k_{c2}}{k_{c3}}\Big)^2-1\Big]P_4\Big] - j\alpha\gamma\Big\}$$

$$+A^{(e)}(\alpha)\Big\{\sinh\gamma_2 d\Big[j\frac{(\alpha\gamma)^2}{\omega\mu_0\gamma_2}\Big[\Big(\frac{k_{c2}}{k_{c3}}\Big)^2-1\Big]P_1 + j\alpha\gamma\frac{\gamma_3}{\gamma_2}\Big(\frac{k_{c2}}{k_{c3}}\Big)^2 P_3$$

$$-j\omega\epsilon_2\gamma_2 P_1\Big] + \cosh\gamma_2 d\Big[j\omega\gamma P_2 - j\omega\epsilon_0\gamma_3\Big(\frac{k_{c2}}{k_{c3}}\Big)^2 P_1$$

$$+j\alpha\gamma\Big[\Big(\frac{k_{c2}}{k_{c3}}\Big)^2-1\Big]P_3\Big] - j\omega\epsilon_0\gamma_1\Big\} = J_z(\alpha)$$

Define the following constants:

$$Q_1 = k_{c2}^2\Big(\frac{\alpha\gamma}{\omega\mu_0\gamma_2}\Big[\Big(\frac{k_{c2}}{k_{c3}}\Big)^2-1\Big]P_2 + \frac{\gamma_3}{\gamma_2}\Big(\frac{k_{c2}}{k_{c3}}\Big)^2 P_4\Big)\sinh\gamma_2 d$$

$$+ k_{c2}^2 P_4\cosh\gamma_2 d + k_{c1}^2 \tag{93}$$

$$Q_2 = k_{c2}^2\Big\{\Big(\frac{\alpha\gamma}{\omega\mu_0\gamma_2}\Big[\Big(\frac{k_{c2}}{k_{c3}}\Big)^2-1\Big]P_1 + \frac{\gamma_3}{\gamma_2}\Big(\frac{k_{c2}}{k_{c3}}\Big)^2 P_3\Big)\sinh\gamma_2 d \tag{94}$$

$$+ P_3\cosh\gamma_2 d\Big\}$$

$$Q_3 = j\Big\{\sinh\gamma_2 d\Big[\omega\epsilon_2\gamma_2 P_2 - \frac{(\alpha\gamma)^2}{\omega\mu_0\gamma_2}\Big[\Big(\frac{k_{c2}}{k_{c3}}\Big)^2-1\Big]P_2$$

$$-\alpha\gamma\frac{\gamma_3}{\gamma_2}\Big(\frac{k_{c2}}{k_{c3}}\Big)^2 P_4\Big] \tag{95}$$

$$+\cosh\gamma_2 d\Big[\omega\epsilon_0\gamma_3\Big(\frac{k_{c2}}{k_{c3}}\Big)^2 P_2 - \alpha\gamma P_4 - \alpha\gamma\Big[\Big(\frac{k_{c2}}{k_{c3}}\Big)^2-1\Big]P_4\Big]$$

$$-\alpha\gamma\Big\}.$$

$$Q_4 = j\Big\{\sinh\gamma_2 d\Big[\frac{(\alpha\gamma)^2}{\omega\mu_0\gamma_2}\Big[\Big(\frac{k_{c2}}{k_{c3}}\Big)^2-1\Big]P_1 + \alpha\gamma\frac{\gamma_3}{\gamma_2}\Big(\frac{k_{c2}}{k_{c3}}\Big)^2 P_3$$

$$-\omega\epsilon_2\gamma_2 P_1\Big]$$

$$+\cosh\gamma_2 d\Big[\alpha\gamma P_2 - \omega\epsilon_0\gamma_3\Big(\frac{k_{c2}}{k_{c3}}\Big)^2 P_1 + \alpha\gamma\Big[\Big(\frac{k_{c2}}{k_{c3}}\Big)^2-1\Big]P_3\Big] \tag{96}$$

$$-\omega\epsilon_0\gamma_1\Big\}.$$

Therefore, equations (91) and (92) become,

$$Q_1 A^{(h)}(\alpha) - Q_2 A^{(e)}(\alpha) = J_x(\alpha) \tag{97}$$

$$Q_3 A^{(h)}(\alpha) + Q_4 A^{(e)}(\alpha) = J_z(\alpha) \tag{98}$$

from where one could obtain expressions for $A^{(e)}(\alpha)$ and $A^{(h)}(\alpha)$ in terms of $J_x(\alpha)$ and $J_z(\alpha)$ as follows,

$$A^{(e)}(\alpha) = \frac{1}{Q_1 Q_4 + Q_2 Q_3}\left[Q_1 J_z(\alpha) - Q_3 J_x(\alpha)\right] \tag{99}$$

$$A^{(h)}(\alpha) = \frac{1}{Q_1 Q_4 + Q_2 Q_3}\left[Q_2 J_z(\alpha) + Q_4 J_x(\alpha)\right] \tag{100}$$

which correspond to equations (B44) and (B45), respectively.

Substituting equations (99) and (100) into equations (B36) and (B37), and solving for $\mathcal{E}_z(\alpha)$ and $\mathcal{E}_x(\quad)$ in terms of $J_z(\alpha)$ and $J_x(\alpha)$, one obtains:

$$\frac{k_{c_1}^2}{Q_1 Q_4 + Q_2 Q_3}\left[Q_1 J_z(\alpha) - Q_3 J_x(\alpha)\right] = \mathcal{E}_z(\alpha) \tag{101}$$

$$\frac{1}{Q_1 Q_4 + Q_2 Q_3}\Big\{-j\alpha\gamma\left[Q_1 J_z(\alpha) - Q_3 J_x(\alpha)\right] + j\omega\mu_0\gamma_1\left[Q_2 J_z(\alpha) + Q_4 J_x(\alpha)\right]\Big\} = \mathcal{E}_x(\alpha) \tag{102}$$

Define the following constants:

$$M_1 = \frac{k_{c_1}^2 Q_1}{Q_1 Q_4 + Q_2 Q_3} \tag{103}$$

$$M_2 = \frac{-k_{c_1}^2 Q_3}{Q_1 Q_4 + Q_2 Q_3} \tag{104}$$

$$M_3 = \frac{j}{Q_1 Q_4 + Q_2 Q_3}\left(\omega\mu_0\gamma_1 Q_2 - \alpha\gamma Q_1\right) \tag{105}$$

$$M_4 = \frac{j}{Q_1 Q_4 + Q_2 Q_3}\left(\omega\mu_0\gamma_1 Q_4 + \alpha\gamma Q_3\right) \tag{106}$$

Then, finally, equations (101) and (102) become,

$$M_1(\alpha,\beta)\, J_z(\alpha) + M_2(\alpha,\beta)\, J_x(\alpha) = \mathcal{E}_z(\alpha) \tag{107}$$

$$M_3(\alpha,\beta)\, J_z(\alpha) + M_4(\alpha,\beta)\, J_x(\alpha) = \mathcal{E}_x(\alpha) \tag{108}$$

which correspond to equations (B46) and (B47), respectively.

APPENDIX E

CURRENT AND FIELD EQUATIONS RE-DEFINITION

From equation (A3), the character of the propagation constant was defined as lossless, i.e.,

$$\gamma = j\beta \ . \tag{109}$$

So, equations (77) through (80) are modified as,

$$N_1 = \left(\frac{k_{c2}}{k_{c3}}\right)^2 \left[\frac{\epsilon_0 \gamma_3}{\epsilon_2 \gamma_2} \left(\frac{k_{c2}}{k_{c3}}\right)^2 \sinh \gamma_2 d + \cosh \gamma_2 d \right] \tag{110}$$

$$N_2 = j \left(\frac{k_{c2}}{k_{c3}}\right)^2 \frac{\alpha\beta}{\omega \epsilon_2 \gamma_2} \left[\left(\frac{k_{c2}}{k_{c3}}\right)^2 - 1 \right] \sinh \gamma_2 d \tag{111}$$

$$N_3 = j \frac{\alpha\beta}{\omega \mu_0 \gamma_1} \left(\frac{k_{c2}}{k_{c3}}\right)^2 \frac{\epsilon_0 \gamma_3}{\epsilon_2 \gamma_2} \left[\left(\frac{k_{c2}}{k_{c3}}\right)^2 - 1 \right] \sinh \gamma_2 d \tag{112}$$

$$N_4 = \frac{1}{\omega \mu_0 \gamma_1} \left\{ \left[\omega \mu_0 \gamma_2 - \frac{(\alpha\beta)^2}{\omega \epsilon_2 \gamma_2} \left[\left(\frac{k_{c2}}{k_{c3}}\right)^2 - 1 \right] \sinh \gamma_2 d \right. \right. \\ \left. + \omega \mu_0 \gamma_3 \left(\frac{k_{c2}}{k_{c3}}\right)^2 \cosh \gamma_2 d \right\} . \tag{113}$$

Consequently, equations (85) through (88) become,

$$P_1 = \frac{N_4}{N_1 N_4 + |N_2||N_3|} \tag{114}$$

$$P_2 = j \frac{|N_2|}{N_1 N_4 + |N_2||N_3|} \tag{115}$$

$$P_3 = j \frac{|N_3|}{N_1 N_4 + |N_2||N_3|} \tag{116}$$

$$P_4 = \frac{N_1}{N_1 N_4 + |N_2||N_3|} \ . \tag{117}$$

Similarly, equations (93) through (96) are re-defined as,

$$Q_1 = k_{c_2}^2 \left\{ \frac{\gamma_3}{\gamma_2} \left(\frac{k_{c2}}{k_{c3}} \right)^2 P_4 - \frac{\alpha\beta}{\omega\mu\gamma_2} \left[\left(\frac{k_{c2}}{k_{c3}} \right)^2 - 1 \right] |P_2| \right\} \sinh \gamma_2 d \tag{118}$$

$$+ k_{c_2}^2 P_4 \cosh \gamma_2 d + k_{c_1}^2$$

$$Q_2 = j\, k_{c_2}^2 \left\{ \left(\frac{\alpha\beta}{\omega\mu_0\gamma_2} \left[\left(\frac{k_{c2}}{k_{c3}} \right)^2 - 1 \right] P_1 + \frac{\gamma_3}{\gamma_2} \left(\frac{k_{c2}}{k_{c3}} \right)^2 |P_3| \right) \sinh \gamma_2 d \right. \tag{119}$$

$$\left. + |P_3| \cosh \gamma_2 d \right\}$$

$$Q_3 = \sinh \gamma_2 d \left\{ \alpha\beta \frac{\gamma_3}{\gamma_2} \left(\frac{k_{c2}}{k_{c3}} \right)^2 P_4 - \omega\epsilon_2 \gamma_2 |P_2| - \frac{(\alpha\beta)^2}{\omega\mu_0\gamma_2} \left[\left(\frac{k_{c2}}{k_{c3}} \right)^2 - 1 \right] |P_2| \right\} \tag{120}$$

$$- \cosh \gamma_2 d \left\{ \omega\epsilon_0 \gamma_3 \left(\frac{k_{c2}}{k_{c3}} \right)^2 |P_2| - \alpha\beta P_4 - \alpha\beta P_4 \left[\left(\frac{k_{c2}}{k_{c3}} \right)^2 - 1 \right] \right\} + \alpha\beta$$

$$Q_4 = -j \left\{ \sinh \gamma_2 d \left[\frac{(\alpha\beta)^2}{\omega\mu_0\gamma_2} \left[\left(\frac{k_{c2}}{k_{c3}} \right)^2 - 1 \right] P_1 + \alpha\beta \frac{\gamma_3}{\gamma_2} |P_3| \left(\frac{k_{c2}}{k_{c3}} \right)^2 \right. \right. \tag{121}$$

$$\left. + \omega\epsilon_2 \gamma_2 P_1 \right] + \cosh \gamma_2 d \left[\alpha\beta |P_3| + \omega\epsilon_0 \gamma_3 \left(\frac{k_{c2}}{k_{c3}} \right)^2 P_1 \right.$$

$$\left. \left. + \alpha\beta \left[\left(\frac{k_{c2}}{k_{c3}} \right)^2 - 1 \right] |P_3| + \omega\epsilon_0 \gamma_1 \right\} .$$

Finally, equations (103) through (106) become,

$$M_1 = \frac{1}{j} \frac{k_{c_1}^2 Q_1}{|Q_2| Q_3 - Q_1 |Q_4|} \tag{122}$$

$$M_2 = \frac{1}{j} \frac{-k_{c_1}^2 Q_3}{|Q_2| Q_3 - Q_1 |Q_4|} \tag{123}$$

$$M_3 = j \frac{1}{|Q_2| Q_3 - Q_1 |Q_4|} \left(\omega\mu_0 \gamma_1 |Q_2| - \alpha\beta Q_1 \right) \tag{124}$$

$$M_4 = j \frac{1}{|Q_2| Q_3 - Q_1 |Q_4|} \left(\alpha\beta Q_3 - \omega\mu_0 \gamma_1 |Q_4| \right) . \tag{125}$$

Furthermore, since from equation (C11) $J_z(\alpha)$ is imaginary and $J_x(\alpha)$ is zero, the expressions for the different constants A through D need to be re-defined.

Equations (B44) and (B45) become,

$$A^{(e)}(\alpha) = \frac{Q_1}{|Q_2|Q_3 - Q_1|Q_4|} \, |J_2(\alpha)| \tag{126}$$

$$A^{(h)}(\alpha) = \frac{j|Q_2|}{|Q_2|Q_3 - Q_1|Q_4|} \, |J_2(\alpha)| \tag{127}$$

which correspond to equations (C16) and (C17), respectively.

Similarly, equations (B42) and (B43) become,

$$C^{(e)}(\alpha) = P_1 A^{(e)}(\alpha) + |P_2| \, |A^{(h)}(\alpha)| \tag{128}$$

$$C^{(h)}(\alpha) = j \left[|P_3| \, A^{(e)}(\alpha) - P_4 \, |A^{(h)}(\alpha)| \right] \tag{129}$$

which correspond, in turn, to equations (C18) and (C19), respectively.

Finally, equations (B38) through (B41) become,

$$B^{(e)}(\alpha) = \frac{1}{\omega\epsilon_2 \gamma_2} \left\{ \omega\epsilon_0 \gamma_3 \left(\frac{k_{c2}}{k_{c3}}\right)^2 C^{(e)}(\alpha) + \alpha\beta \left[\left(\frac{k_{c2}}{k_{c3}}\right)^2 - 1 \right] |C^{(h)}(\alpha)| \right\} \tag{130}$$

$$B^{(h)}(\alpha) = j \frac{1}{\omega\mu_0 \gamma_2} \left\{ \alpha\beta \left[\left(\frac{k_{c2}}{k_{c3}}\right)^2 - 1 \right] C^{(e)}(\alpha) + \omega\mu_0 \gamma_3 \left(\frac{k_{c2}}{k_{c3}}\right)^2 |C^{(h)}(\alpha)| \right\} \tag{131}$$

$$D^{(e)}(\alpha) = \left(\frac{k_{c2}}{k_{c3}}\right)^2 C^{(e)}(\alpha) \tag{132}$$

$$D^{(h)}(\alpha) = j \left(\frac{k_{c2}}{k_{c3}}\right)^2 |C^{(h)}(\alpha)| \tag{133}$$

which correspond to equations (C20) through (C23), respectively.

APPENDIX F - DISPERSION CHARACTERISTIC PROGRAM

```
C
C
C     THIS PROGRAM SOLVES THE DISPERSION CHARACTERISTICS OF
C     COPLANAR PARALLEL STRIPS ON A DIELECTRIC SUBSTRATE.
C
C     PROGRAM ASSUMES FOLLOWING PARAMETERS ARE KNOWN
C       - THICKNESS OF THE SLAB,DD,IN MILLIMETERS
C       - WIDTH OF THE STRIPS,WW,IN MILLIMETERS
C       - DIELECTRIC'S RELATIVE PERMITTIVITY,ER
C       - RATIO OF SEPARATION BETWEEN CONDUCTORS TO WIDTH OF
C         CONDUCTORS,SOW
C       - RATIO OF SUBSTRATE'S THICKNESS TO FREE-SPACE
C         WAVELENGTH,DRATIO
C
C     THE METHOD USED IN FINDING THE EFFECTIVE WAVELENGTH IN
C     THE STRUCTURE,LPRIME,IS AN ITERATION METHOD,WHICH
C     SOLVES FOR THE ZERO VALUE OF AN INTEGRAL IN THE ALFA-
C     DOMAIN FOR PROGRESSIVELY LARGER VALUES OF EFFECTIVE
C     WAVELENGTH.
C
C     THE SUMMATION IN ALFA IS AN APPROXIMATION TO AN
C     INTEGRATION IN THE ALFA-DOMAIN FROM (-INFINITY) TO
C     (+INFINITY).
C
C     THE APPROXIMATIONS USED IN THE PRESENT PROGRAM ARE
C       - THE ITERATION STEP,X
C       - THE EPSILON VALUE,DELTA
C       - THE INTEGRATION STEP,A
C       - THE LIMITS OF THE ALFA-DOMAIN INTEGRATION,ALFA
C         AND B.
C     IT IS CLEAR THAT ANY OF THESE VALUES MAY BE CHANGED
C     ACCORDING TO THE SPECIFIC CASE OR DESIRED ACCURACY.
C
C     THE QUANTITY EO IS THE FREE-SPACE PERMITTIVITY IN MKS
C     UNITS.
C     THE QUANTITY MU IS THE FREE-SPACE PERMEABILITY IN MKS
C     UNITS.
C     THE QUANTITY C IS THE FREE-SPACE SPEED OF LIGHT IN
C     METERS PER SECOND.
C     THE QUANTITY M1 IS THE STRIP CURRENT IN AMPERES.
C
C     THE VECTOR SOW STORES THE RATIOS OF SEPARATION OF
C     CONDUCTORS TO WIDTH OF CONDUCTORS WHICH ARE DESIRED
C     TO SOLVE FOR. THE NUMBER J SETS THE DIMENSION OF THIS
C     VECTOR.
C
C     PROGRAM DEVELOPED BY LT(JG) ARMANDO LUNA ECHEANDIA,
C     PERUVIAN NAVY.
C     SUPERVISOR - PROF. JEFFREY B. KNORR,PH.D.
C     U.S. NAVAL POSTGRADUATE SCHOOL,MONTEREY,CALIFORNIA
C     SEPTEMBER,1973.
C
C
      IMPLICIT REAL*4 (M,N,K,L)
      DIMENSION SOW(7)
      DATA WW/3.17/,DD/3.17/,ER/12.0/
      DATA A/0.5/
      DATA B/1700./
      DATA X/.4E-4/,DELTA/1.0E-7/
      J=7
      PI=3.14159
      EO=1.E-9/(36.*PI)
      MU=4.E-7*PI
      C=3.E8
      FACTOR=1.E9
      W=WW*1.E-3
      D=DD*1.E-3
      READ (5,100) (SOW(I),I=1,J)
  100 FORMAT (7F5.3)
   24 READ (5,18) DRATIO
   18 FORMAT (F6.4)
```

```
      FREQ=DRATIO*C/D
      FRE=FREQ/FACTOR
      LPRIME=C/(FREQ*SQRT(ER))
      OMEGA=2.*PI*FREQ
      OE=OMEGA*EO*ER
      OM=OMEGA*MU
      K1=OMEGA*SQRT(MU*EO)
      K2=K1*SQRT(ER)
      K3=K1
      K11=K1**2
      K22=K2**2
      DO 101  I=1,J
      S=SOW(I)
   22 BETA=2.*PI/LPRIME
      B2=BETA**2
      KC1SQ=K11-B2
      KC2SQ=K22-B2
      KC3SQ=KC1SQ
      KC4=(KC2SQ/KC3SQ)
      KC5=KC4-1.0
      KC6=KC5**2
      M3=0.0
      ALFA=-1700.
    1 ALPHA=ALFA+A/2.0
      A2=ALPHA**2
C
C     PHYSICAL CONFIGURATION DEPENDENCE AS DERIVED BY THE
C     FOURIER TRANSFORM OF THE CURRENT DENSITIES.
C
      F1=ALPHA*(S+W)/2.0
      F2=ALPHA*W/2.0
      S1=SIN(F1)
      S2=SIN(F2)
      S3=S1**2
      S4=S2**2
      AB=ALPHA*BETA
      G1=SQRT(A2+B2-K11)
      G3=G1
      G11=G1**2
      G22=A2+B2-K22
      IF (G22.LT.0.0) GO TO 7
C
C     HYPERBOLIC CASE, GAMMA 2 IS REAL
C
      G2=SQRT(G22)
      G2D=G2*D
      SI=SINH(G2D)
      CO=COSH(G2D)
C
C     INTERMEDIATE ALGEBRAIC STEPS
C
      N1=KC4*(G3/(G2*ER)*KC4*SI+CO)
      N2=KC4*AB/(OE*G2)*KC5*SI
      N3=AB/(OM*G1)*KC4*G3/(G2*ER)*KC5*SI
      N4=1./(OM*G1)*((OM*G2-(AB**2)*KC6/(OE*G2))*SI+
     1OM*G3*KC4*CO)
      N5=N1*N4+N2*N3
      P1=N4/N5
      P2=N2/N5
      P3=N3/N5
      P4=N1/N5
      Q1=KC2SQ*((G3/G2)*KC4*P4-AB/(OM*G2)*P2*KC5*SI+KC2SQ*
     1P4*CO+KC1SQ
      Q2=KC2SQ*((AB/(OM*G2)*KC5*P1+(G3/G2)*KC4*P3)*SI+P3*CO)
      Q3=(AB*(G3/G2)*KC4*P4-OE*G2*P2-(AB**2)/(OM*G2)*KC5*
     1P2)*SI-((OE/ER)*G3*KC4*P2-AB*P4-AB*P4*KC5)*CO+AB
      Q4=((AB**2)/(OM*G2)*KC5*P1+AB*(G3/G2)*P3*KC4+OE*G2*
     1P1)*SI+(AB*P3+(OE/ER)*G3*KC4*P1+AB*KC5*P3)*CO+(OE/ER)*
     2G1
      GO TO 9
C
C     TRIGONOMETRIC CASE, GAMMA 2 IS IMAGINARY
```

```
C
    7 G22=ABS(A2+B2-K22)
      G2=SQRT(G22)
      G2D=G2*D
      TSI=SIN(G2D)
      TCO=COS(G2D)
C
C     INTERMEDIATE ALGABRAIC STEPS
C
      N1=KC4*(G3/(G2*ER)*KC4*TSI+TCO)
      N2=KC4*AB/(OE*G2)*KC5*TSI
      N3=AB/(OM*G1)*KC4*G3/(G2*ER)*KC5*TSI
      N4=1./(OM*G1)*((-OM*G2-(AB**2)*KC6/(OE*G2))*TSI+
     1OM*G3*KC4*TCO)
      N5=N1*N4+N2*N3
      P1=N4/N5
      P2=N2/N5
      P3=N3/N5
      P4=N1/N5
      Q1=KC2SQ*((G3/G2)*KC4*P4-AB/(OM*G2)*P2*KC5)*TSI+
     1KC2SQ*P4*TCO+KC1SQ
      Q2=KC2SQ*((AB/(OM*G2)*KC5*P1+(G3/G2)*KC4*P3)*TSI+
     1P3*TCO)
      Q3=(AB*(G3/G2)*KC4*P4+OE*G2*P2-(AB**2)/(OM*G2)*KC5*
     1P2)*TSI-((OE/ER)*G3*KC4*P2-AB*P4-AB*P4*KC5)*TCO+AB
      Q4=((AB**2)/(OM*G2)*KC5*P1+AB*(G3/G2)*P3*KC4-OE*G2*
     1P1)*TSI+(AB*P3+(OE/ER)*G3*KC4*P1+AB*KC5*P3)*TCO+
     2(OE/ER)*G1
    9 Q5=Q2*Q3-Q1*Q4
      M1=KC1SQ*Q1/Q5
      M2=16.*M1/A2*S3*S4
      M3=M3+M2*A
      ALFA=ALFA+A
      IF (ALFA.GT.B) GO TO 2
      GO TO 1
C
C     ITERATION STEP
C
    2 IF (M3.LT.DELTA) GO TO 19
      LPRIME=LPRIME+X
      GO TO 22
   19 WRITE (6,191) S,FREQ,LPRIME
  191 FORMAT(' ',/,3X,'LAMBDA PRIME FOR S = ',F10.5,2X,
     1'AND FREQ = ',E16.7,2X,'IS ',E16.7,/)
  101 CONTINUE
      GO TO 24
   23 STOP
      END
```

APPENDIX G

CHARACTERISTIC IMPEDANCE RELATED PARAMETERS

It is convenient to recall the Hertzian potential functions for the three regions of analysis. The electric field functions are,

$$\Phi_1^{(e)}(\alpha,y) = A^{(e)}(\alpha)\, e^{-\gamma_1(y-d)} \tag{134}$$

$$\Phi_2^{(e)}(\alpha,y) = B^{(e)}(\alpha) \sinh \gamma_2 y + C^{(e)}(\alpha) \cosh \gamma_2 y \tag{135}$$

$$\Phi_3^{(e)}(\alpha,y) = D^{(e)}(\alpha)\, e^{\gamma_3 y} \tag{136}$$

from where the corresponding y-derivatives can be obtained as,

$$\frac{\partial \Phi_1^{(e)}}{\partial y} = -\gamma_1 A^{(e)}(\alpha)\, e^{-\gamma_1(y-d)} \tag{137}$$

$$\frac{\partial \Phi_2^{(e)}}{\partial y} = \gamma_2 \left[B^{(e)}(\alpha) \cosh \gamma_2 y + C^{(e)}(\alpha) \sinh \gamma_2 y \right] \tag{138}$$

$$\frac{\partial \Phi_3^{(e)}}{\partial y} = \gamma_3 D^{(e)}(\alpha)\, e^{\gamma_3 y} \tag{139}$$

As stated in the dispersion characteristic part of the present work, specifically in equations (C16) through (C23), the magnetic field related constants have imaginary character. Therefore, it is convenient to modify the magnetic field Hertzian potential functions to account for this fact.

$$\Phi_1^{(h)}(\alpha,y) = j\, |A^{(h)}(\alpha)|\, e^{-\gamma_1(y-d)} \tag{140}$$

$$\Phi_2^{(h)}(\alpha,y) = j \left[|B^{(h)}(\alpha)| \sinh \gamma_2 y + |C^{(h)}(\alpha)| \cosh \gamma_2 y \right] \tag{141}$$

$$\Phi_3^{(h)}(\alpha,y) = j\, |D^{(h)}(\alpha)|\, e^{\gamma_3 y} \tag{142}$$

and, again, the corresponding y-derivatives can be obtained as,

$$\frac{\partial \Phi_1^{(h)}}{\partial y} = -j\gamma_1 |A^{(h)}(\alpha)| e^{-\gamma_1(y-d)} \tag{143}$$

$$\frac{\partial \Phi_2^{(h)}}{\partial y} = j\gamma_2 \left[|B^{(h)}(\alpha)| \cosh \gamma_2 y + |C^{(h)}(\alpha)| \sinh \gamma_2 y \right] \tag{144}$$

$$\frac{\partial \Phi_3^{(h)}}{\partial y} = j\gamma_3 |D^{(h)}(\alpha)| e^{\gamma_3 y} . \tag{145}$$

Since an integration in the y-domain comprises three well-defined regions, one should separate the y-integral into three complementary integrals, one per each region.

Therefore, analyzing each term in the average power expression of equation (G12), one obtains,

$$\Phi_1^{(e)}(\alpha,y)\, \Phi_1^{(e)*}(\alpha,y) = |A^{(e)}(\alpha)|^2 e^{-2\gamma_1(y-d)} \tag{146}$$

$$\begin{aligned}\Phi_2^{(e)}(\alpha,y)\, \Phi_2^{(e)*}(\alpha,y) &= |B^{(e)}(\alpha)|^2 \sinh^2 \gamma_2 y + |C^{(e)}(\alpha)|^2 \cosh^2 \gamma_2 y \\ &\quad + 2B^{(e)}(\alpha)\, C^{(e)}(\alpha) \sinh \gamma_2 y \cosh \gamma_2 y\end{aligned} \tag{147}$$

$$\Phi_3^{(e)}(\alpha,y)\, \Phi_3^{(e)*}(\alpha,y) = |D^{(e)}(\alpha)|^2 e^{2\gamma_3 y} \tag{148}$$

$$\Phi_1^{(h)}(\alpha,y)\, \Phi_1^{(h)*}(\alpha,y) = |A^{(h)}(\alpha)|^2 e^{-2\gamma_1(y-d)} \tag{149}$$

$$\begin{aligned}\Phi_2^{(h)}(\alpha,y)\, \Phi_2^{(h)*}(\alpha,y) &= |B^{(h)}(\alpha)|^2 \sinh^2 \gamma_2 y + |C^{(h)}(\alpha)|^2 \cosh^2 \gamma_2 y \\ &\quad + 2|B^{(h)}(\alpha)||C^{(h)}(\alpha)| \sinh \gamma_2 y \cosh \gamma_2 y\end{aligned} \tag{150}$$

$$\Phi_3^{(h)}(\alpha,y)\, \Phi_3^{(h)*}(\alpha,y) = |D^{(h)}(\alpha)|^2 e^{2\gamma_3 y} \tag{151}$$

$$\frac{\partial \Phi_1^{(e)}}{\partial y} \frac{\partial \Phi_1^{(e)*}}{\partial y} = \gamma_1^2 |A^{(e)}(\alpha)|^2 e^{-2\gamma_1(y-d)} \tag{152}$$

$$\begin{aligned}\frac{\partial \Phi_2^{(e)}}{\partial y} \frac{\partial \Phi_2^{(e)*}}{\partial y} &= \gamma_2^2 \Big[|B^{(e)}(\alpha)|^2 \cosh^2 \gamma_2 y + |C^{(e)}(\alpha)|^2 \sinh^2 \gamma_2 y \\ &\quad + 2B^{(e)}(\alpha)\, C^{(e)}(\alpha) \sinh \gamma_2 y \cosh \gamma_2 y \Big]\end{aligned} \tag{153}$$

$$\frac{\partial \Phi_3^{(e)}}{\partial y} \frac{\partial \Phi_3^{(e)*}}{\partial y} = \gamma_3^2 \, |D^{(e)}(\alpha)|^2 \, e^{2\gamma_3 y} \tag{154}$$

$$\frac{\partial \Phi_1^{(h)}}{\partial y} \frac{\partial \Phi_1^{(h)*}}{\partial y} = \gamma_1^2 \, |A^{(h)}(\alpha)|^2 \, e^{-2\gamma_1 (y-d)} \tag{155}$$

$$\begin{aligned} \frac{\partial \Phi_2^{(h)}}{\partial y} \frac{\partial \Phi_2^{(h)*}}{\partial y} = \gamma_2^2 \Big[& |B^{(h)}(\alpha)|^2 \cosh^2 \gamma_2 y + |C^{(h)}(\alpha)|^2 \sinh^2 \gamma_2 y \\ & + 2 |B^{(h)}(\alpha)| |C^{(h)}(\alpha)| \sinh \gamma_2 y \cosh \gamma_2 y \Big] \end{aligned} \tag{156}$$

$$\frac{\partial \Phi_3^{(h)}}{\partial y} \frac{\partial \Phi_3^{(h)*}}{\partial y} = \gamma_3^2 \, |D^{(h)}(\alpha)|^2 e^{2\gamma_3 y} \tag{157}$$

$$j \, \Phi_1^{(e)}(\alpha, y) \frac{\partial \Phi_1^{(h)*}}{\partial y} = -\gamma_1 \, A^{(e)}(\alpha) \, |A^{(h)}(\alpha)| \, e^{-2\gamma_1 (y-d)} \tag{158}$$

$$\begin{aligned} j \, \Phi_2^{(e)}(\alpha, y) \frac{\partial \Phi_2^{(h)*}}{\partial y} = \gamma_2 \Big[& B^{(e)}(\alpha) |C^{(h)}(\alpha)| \sinh^2 \gamma_2 y + |B^{(h)}(\alpha)| \, C^{(e)}(\alpha) \cosh^2 \gamma_2 y \\ & + \left(B^{(e)}(\alpha) \, |B^{(h)}(\alpha)| + C^{(e)}(\alpha) \, |C^{(h)}(\alpha)| \right) \sinh \gamma_2 y \cosh \gamma_2 y \Big] \end{aligned} \tag{159}$$

$$j \, \Phi_3^{(e)}(\alpha, y) \frac{\partial \Phi_3^{(h)*}}{\partial y} = \gamma_3 \, D^{(e)}(\alpha) \, |D^{(h)}(\alpha)| \, e^{2\gamma_3 y} \tag{160}$$

$$j \, \frac{\partial \Phi_1^{(h)}}{\partial y} \Phi_1^{(e)*}(\alpha, y) = \gamma_1 \, A^{(e)}(\alpha) \, |A^{(h)}(\alpha)| \, e^{-2\gamma_1 (y-d)} \tag{161}$$

$$\begin{aligned} j \, \frac{\partial \Phi_2^{(h)}}{\partial y} \Phi_2^{(e)*}(\alpha, y) = -\gamma_2 \Big[& |B^{(h)}(\alpha)| C^{(e)}(\alpha) \cosh^2 \gamma_2 y + B^{(e)}(\alpha) \, |C^{(h)}(\alpha)| \sinh^2 \gamma_2 y \\ & + \left(B^{(e)}(\alpha) \, |B^{(h)}(\alpha)| + C^{(e)}(\alpha) \, |C^{(h)}(\alpha)| \right) \sinh \gamma_2 y \cosh \gamma_2 y \Big] \end{aligned} \tag{162}$$

$$j \, \frac{\partial \Phi_3^{(h)}}{\partial y} \Phi_3^{(e)*}(\alpha, y) = -\gamma_3 \, D^{(e)}(\alpha) \, |D^{(h)}(\alpha)| \, e^{2\gamma_3 y} \tag{163}$$

$$j \, \frac{\partial \Phi_1^{(e)}}{\partial y} \Phi_1^{(h)*}(\alpha, y) = -\gamma_1 \, A^{(e)}(\alpha) \, |A^{(h)}(\alpha)| \, e^{-2\gamma_1 (y-d)} \tag{164}$$

$$\begin{aligned} j \, \frac{\partial \Phi_2^{(e)}}{\partial y} \Phi_2^{(h)*}(\alpha, y) = \gamma_2 \Big[& B^{(e)}(\alpha) |C^{(h)}(\alpha)| \cosh^2 \gamma_2 y + |B^{(h)}(\alpha)| \, C^{(e)}(\alpha) \sinh^2 \gamma_2 y \\ & + \left(B^{(e)}(\alpha) |B^{(h)}(\alpha)| + C^{(e)}(\alpha) \, |C^{(h)}(\alpha)| \right) \sinh \gamma_2 y \cosh \gamma_2 y \Big] \end{aligned} \tag{165}$$

$$j \, \frac{\partial \Phi_3^{(e)}}{\partial y} \Phi_3^{(h)*}(\alpha, y) = \gamma_3 \, D^{(e)}(\alpha) \, D^{(h)}(\alpha) \, e^{2\gamma_3 y} \tag{166}$$

$$j\,\Phi_1^{(h)}(\alpha,y)\,\frac{\partial \Phi_1^{(e)*}}{\partial y} = \gamma_1 A^{(e)}(\alpha)\,|A^{(h)}(\alpha)|\,e^{-2\gamma_1(y-d)} \tag{167}$$

$$\begin{aligned} j\,\Phi_2^{(h)}(\alpha,y)\,\frac{\partial \Phi_2^{(e)*}}{\partial y} = -\gamma_2 \Big[& B^{(e)}(\alpha)\,|C^{(h)}(\alpha)| \cosh^2 \gamma_2 y + |B^{(h)}(\alpha)|\,C^{(e)}(\alpha) \sinh^2 \gamma_2 y \\ & + \left(B^{(e)}(\alpha)\,|B^{(h)}(\alpha)| + C^{(e)}(\alpha)\,|C^{(h)}(\alpha)|\right) \sinh \gamma_2 y \cosh \gamma_2 y \Big] \end{aligned} \tag{168}$$

$$j\,\Phi_3^{(h)}(\alpha,y)\,\frac{\partial \Phi_3^{(e)*}}{\partial y} = -\gamma_3 D^{(e)}(\alpha)\,|D^{(h)}(\alpha)|\,e^{2\gamma_3 y} \,. \tag{169}$$

Therefore, the expression for average power in equation (G12) becomes,

$$\begin{aligned} (\alpha,y) = \frac{1}{4\pi} \iint_{-\infty}^{+\infty} \Big\{ & \omega\beta(\alpha^2-\gamma_1^2)\left(\epsilon_0 |A^{(e)}(\alpha)|^2 + \mu_0 |A^{(h)}(\alpha)|^2\right) e^{-2\gamma_1(y-d)} \\ & + \omega\beta(\alpha^2-\gamma_3^2)\left(\epsilon_0 |D^{(e)}(\alpha)|^2 + \mu_0 |D^{(h)}(\alpha)|^2\right) e^{2\gamma_3 y} \\ \sinh^2 \gamma_2 y \Big[& \alpha^2\beta\omega\left(\epsilon_2 |B^{(e)}(\alpha)|^2 + \mu_0 |B^{(h)}(\alpha)|^2\right) - \omega\beta\gamma_2^2\left(\epsilon_2 |C^{(e)}(\alpha)|^2 + \mu_0 |C^{(h)}(\alpha)|^2\right) \\ & + \alpha\beta^2\gamma_2\left(|B^{(h)}(\alpha)|\,C^{(e)}(\alpha) - B^{(e)}(\alpha)\,|C^{(h)}(\alpha)|\right) + \alpha k_2^2 \gamma_2 \left(B^{(e)}(\alpha)\,|C^{(h)}(\alpha)| - |B^{(h)}(\alpha)|\,C^{(e)}(\alpha)\right) \Big] \\ \cosh^2 \gamma_2 y \Big[& \alpha^2\beta\omega\left(\epsilon_2 |C^{(e)}(\alpha)|^2 + \mu_0 |C^{(h)}(\alpha)|^2\right) - \omega\beta\gamma_2^2\left(\epsilon_2 |B^{(e)}(\alpha)|^2 + \mu_0 |B^{(h)}(\alpha)|^2\right) \\ & + \alpha\beta^2\gamma_2\left(B^{(e)}(\alpha)\,|C^{(h)}(\alpha)| - |B^{(h)}(\alpha)|\,C^{(e)}(\alpha)\right) + \alpha k_2^2 \gamma_2 \left(|B^{(h)}(\alpha)|\,C^{(e)}(\alpha) - B^{(e)}(\alpha)\,|C^{(h)}(\alpha)|\right) \Big] \\ \sinh 2\gamma_2 y \Big[& \omega\beta\left(\epsilon_2 B^{(e)}(\alpha)\,C^{(e)}(\alpha) + \mu_0 |B^{(h)}(\alpha)|\,|C^{(h)}(\alpha)|\right)(\alpha^2-\gamma_2^2) \Big] \Big\}\, d\alpha\, dy \end{aligned} \tag{170}$$

and, for the case where γ_2 is imaginary, the following relations hold,

$$\gamma_2 = j\gamma_2''$$

$$\sinh \gamma_2 y = j \sin \gamma_2'' y$$

$$\cosh \gamma_2 y = \cos \gamma_2'' y$$

$$\sinh^2 \gamma_2 y = -\sin^2 \gamma_2'' y$$

$$\cosh^2 \gamma_2 y = \cos^2 \gamma_2'' y$$

$$\sinh 2\gamma_2 y = j \sin 2\gamma_2'' y \,.$$

Therefore, equation (170) becomes,

$$
\begin{aligned}
P_{AVE}(\alpha, y) = \frac{1}{4\pi} \iint_{-\infty}^{+\infty} \Big\{ & \omega\beta(\alpha^2 - \gamma_1^2)\left(\epsilon_0 |A^{(e)}(\alpha)|^2 + \mu_0 |A^{(h)}(\alpha)|^2\right) e^{-2\gamma_1(y-d)} \\
& + \omega\beta(\alpha^2 - \gamma_3^2)\left(\epsilon_0 |D^{(e)}(\alpha)|^2 + \mu_0 |D^{(h)}(\alpha)|^2\right) e^{2\gamma_3 y} \\
& - \sin^2 \gamma_2'' y \Big[\alpha^2\beta\omega\left(\epsilon_2 |B^{(e)}(\alpha)|^2 + \mu_0 |B^{(h)}(\alpha)|^2\right) + \omega\beta\gamma_2''^2\left(\epsilon_2 |C^{(e)}(\alpha)|^2 + \mu_0 |C^{(h)}(\alpha)|^2\right) \\
& \quad - \alpha\beta^2\gamma_2''\left(|B^{(h)}(\alpha)| C^{(e)}(\alpha) - B^{(e)}(\alpha) |C^{(h)}(\alpha)|\right) \\
& \quad - \alpha k_2^2 \gamma_2''\left(B^{(e)}(\alpha) |C^{(h)}(\alpha)| - |B^{(h)}(\alpha)| C^{(e)}(\alpha)\right)\Big] \\
& + \cos^2 \gamma_2'' y \Big[\alpha^2\beta\omega\left(\epsilon_2 |C^{(e)}(\alpha)|^2 + \mu_0 |C^{(h)}(\alpha)|^2\right) + \omega\beta\gamma_2''^2\left(\epsilon_2 |B^{(e)}(\alpha)|^2 + \mu_0 |B^{(h)}(\alpha)|^2\right) \\
& \quad - \alpha\beta^2\gamma_2''\left(B^{(e)}(\alpha) |C^{(h)}(\alpha)| - |B^{(h)}(\alpha)| C^{(e)}(\alpha)\right) \\
& \quad - \alpha k_2^2 \gamma_2''\left(|B^{(h)}(\alpha)| C^{(e)}(\alpha) - B^{(e)}(\alpha) |C^{(h)}(\alpha)|\right)\Big]\Big\} \, d\alpha \, dy.
\end{aligned}
\tag{170a}
$$

These two expressions correspond to equations (G13) and (G13a), respectively.

The y-dependence of the average power disappears through corresponding integrations on regions 1, 2, and 3, as indicated below:

- Region 1,

$$\int_d^{\infty} e^{-2\gamma_1(y-d)} \, dy = \frac{1}{2\gamma_1} . \tag{171}$$

- Region 2,

$$\int_0^d \sinh^2 \gamma_2 y \, dy = \frac{1}{4\gamma_2} \sinh 2\gamma_2 d - \frac{1}{2} d \tag{172}$$

$$\int_0^d \cosh^2 \gamma_2 y \, dy = \frac{1}{4\gamma_2} \sinh 2\gamma_2 d + \frac{1}{2} d \tag{173}$$

$$\int_0^d \sinh 2\gamma_2 y \, dy = \frac{1}{2\gamma_2} (\cosh 2\gamma_2 d - 1) \tag{174}$$

$$\int_0^d \sin^2 \gamma_2'' y \, dy = \frac{1}{2} d - \frac{1}{4\gamma_2''} \sin 2\gamma_2'' d \tag{175}$$

$$\int_0^d \cos^2 \gamma_2'' y \, dy = \frac{1}{2} d + \frac{1}{4\gamma_2''} \sin 2\gamma_2'' d \tag{176}$$

$$\int_{0}^{d} \sin 2\gamma_2'' y \, dy = \frac{1}{2\gamma_2''} \left(1 - \cos 2\gamma_2'' d\right) . \tag{177}$$

- Region 3,

$$\int_{-\infty}^{0} e^{2\gamma_3 y} dy = \frac{1}{2\gamma_3} . \tag{178}$$

Substituting equations (171) through (178) into equations (170) and (170a), one obtains equations (G14) and (G14a).

APPENDIX H - CHARACTERISTIC IMPEDANCE PROGRAM

```
C
C
C     THIS PROGRAM SOLVES THE CHARACTERISTIC IMPEDANCE OF
C     COPLANAR PARALLEL STRIPS ON A DIELECTRIC SUBSTRATE.
C
C     PROGRAM ASSUMES FOLLOWING PARAMETERS ARE KNOWN
C        - THICKNESS OF THE SLAB,DD,IN MILLIMETERS
C        - WIDTH OF THE STRIPS,WW,IN MILLIMETERS
C        - DIELECTRIC'S RELATIVE PERMITTIVITY,ER
C        - RATIO OF SEPARATION BETWEEN CONDUCTORS TO WIDTH OF
C          CONDUCTORS,SOW
C        - RATIO OF SUBSTRATE'S THICKNESS TO FREE-SPACE
C          WAVELENGTH,DRATIO
C        - RATIO OF EFFECTIVE DIELECTRIC'S WAVELENGTH TO FREE
C          SPACE WAVELENGTH,LRATIO
C
C     THE METHOD USED IN FINDING THE CHARACTERISTIC
C     IMPEDANCE IS A DIRECT SUMMATION OF INFINITESIMAL
C     VALUES BY A MODIFIED SIMPSON'S RULE.
C
C     THE SUMMATION IN ALFA IS AN APPROXIMATION TO AN
C     INTEGRATION IN THE ALFA-DOMAIN FROM (-INFINITY) TO
C     (+INFINITY).
C
C     THE APPROXIMATIONS USED IN THE PRESENT PROGRAM ARE
C        - THE INTEGRATION STEP,A
C        - THE LIMITS OF THE ALFA-DOMAIN INTEGRATION,ALFA
C          AND B.
C     IT IS CLEAR THAT ANY OF THESE VALUES MAY BE CHANGED
C     ACCORDING TO THE SPECIFIC CASE OR DESIRED ACCURACY.
C
C     THE QUANTITY EO IS THE FREE-SPACE PERMITTIVITY IN MKS
C     UNITS.
C     THE QUANTITY MU IS THE FREE-SPACE PERMEABILITY IN MKS
C     UNITS.
C     THE QUANTITY C IS THE FREE-SPACE SPEED OF LIGHT IN
C     METERS PER SECOND.
C     THE QUANTITY M1 IS THE STRIP CURRENT IN AMPERES.
C
C     THE NUMBER JJ SETS THE NUMBER OF DRATIO VALUES.
C     THE NUMBER OF IL SETS THE NUMBER OF SOW VALUES.
C
C     PROGRAM DEVELOPED BY LT(JG) ARMANDO LUNA ECHEANDIA,
C     PERUVIAN NAVY.
C     SUPERVISOR - PROF. JEFFREY B. KNORR,PH.D.
C     U.S. NAVAL POSTGRADUATE SCHOOL,MONTEREY,CALIFORNIA
C     SEPTEMBER,1973.
C
C
      IMPLICIT REAL*4 (M,N,K,L)
      DATA WW/3.17/,DD/3.17/,ER/20./
      DATA A/0.5/
      DATA B/1700./
      IL=7
      JJ=6
      PI=3.14159
      FPI=4.*PI
      EO=1.E-9/(36.*PI)
      MU=4.E-7*PI
      C=3.E8
      FACTOR=1.E9
      W=WW*1.E-3
      D=DD*1.E-3
      M1=W**2
      DO 11 I=1,IL
      READ (5,101) SOW
  101 FORMAT (F5.3)
      S=SOW*W
      DO 13 J=1,JJ
      READ (5,102) DRATIO,LRATIO
  102 FORMAT (F6.4,F8.6)
```

```
      FREQ=DRATIO*C/D
      FRE=FREQ/FACTOR
      LAMBDA=C/FREQ
      LPRIME=LRATIO*LAMBDA
      OMEGA=2.*PI*FREQ
      OE=OMEGA*EO*ER
      OM=OMEGA*MU
      K1=OMEGA*SQRT(MU*EO)
      K2=K1*SQRT(ER)
      K3=K1
      K11=K1**2
      K22=K2**2
   22 BETA=2.*PI/LPRIME
      B2=BETA**2
      KC1SQ=K11-B2
      KC2SQ=K22-B2
      KC3SQ=KC1SQ
      KC4=(KC2SQ/KC3SQ)
      KC5=KC4-1.0
      KC6=KC5**2
      M3=0.0
      ALFA=-1700.
    1 ALPHA=ALFA+A/2.0
      A2=ALPHA**2
C
C     PHYSICAL CONFIGURATION DEPENDENCE AS DERIVED BY THE
C     FOURIER TRANSFORM OF THE CURRENT DENSITIES.
C
      F1=ALPHA*(S+W)/2.0
      F2=ALPHA*W/2.0
      S1=SIN(F1)
      S2=SIN(F2)
      CJ=(4./ALPHA)*S1*S2
      AB=ALPHA*BETA
      G1=SQRT(A2+B2-K11)
      G3=G1
      ALG1=A2/G1-G1
      G11=G1**2
      G22=A2+B2-K22
      IF (G22.LT.0.0) GO TO 7
C
C     HYPERBOLIC CASE, GAMMA 2 IS REAL
C
      G2=SQRT(G22)
      G2D=G2*D
      G22D=2.*G2D
      ALG2=A2/G2-G2
      ALAG2=A2+G22
      SI=SINH(G2D)
      CC=COSH(G2D)
      SI2=SINH(G22D)
      FCO=COSH(G22D)-1.
C
C     INTERMEDIATE ALGEBRAIC STEPS
C
      N1=KC4*(G3/(G2*ER)*KC4*SI+CO)
      N2=KC4*AB/(OE*G2)*KC5*SI
      N3=AB/(OM*G1)*KC4*G3/(G2*ER)*KC5*SI
      N4=1./(OM*G1)*((OM*G2-(AB**2)*KC6/(OE*G2))*SI+OM*G0*KC
     14*CO)
      N5=N1*N4+N2*N3
      P1=N4/N5
      P2=N2/N5
      P3=N3/N5
      P4=N1/N5
      Q1=KC2SQ*((G3/G2)*KC4*P4-AB/(OM*G2)*P2*KC5)*SI+KC2SQ*P
     14*CO+KC1SQ
      Q2=KC2SQ*((AB/(OM*G2)*KC5*P1+(G3/G2)*KC4*F3)*SI+P3*CC)
      Q3=(AB*(G3/G2)*KC4*P4-OE*G2*P2-(AB**2)/(OM*G2)*KC5*P2)
     1*SI-((OE/ER)*G3*KC4*F2-AB*P4-AB*P4*KC5)*CO+AB
      Q4=((AB**2)/(OM*G2)*KC5*P1+AB*(G3/G2)*P3*KC4+OE*G2*P1)
     1*SI+(AB*P3+(OE/ER)*G3*KC4*P1+AB*KC5*P3)*CC+(OE/ER)*G1
```

```
      Q5=Q2*Q3-Q1*Q4
      AE=Q1/Q5*CJ
      AH=Q2/Q5*CJ
      CE=P1*AE+P2*AH
      CH=P3*AE-P4*AH
      BE=1./(OE*G2)*((OE/ER)*G3*KC4*CE+AB*KC5*CH)
      BH=1./(OM*G2)*(AB*KC5*CE+OM*G3*KC4*CH)
      DE=KC4*CE
      DH=KC4*CH
      AE2=AE**2
      AH2=AH**2
      BE2=BE**2
      BH2=BH**2
      CE2=CE**2
      CH2=CH**2
      DE2=DE**2
      DH2=DH**2
      T1=(AE2+DE2)*EO
      T2=(AH2+DH2)*MU
      T3=(BE2+CE2)*EO*ER
      T4=(BH2+CH2)*MU
      T5=(CE2-BE2)*EO*ER
      T6=(CH2-BH2)*MU
      T7=BE*CH-BH*CE
      T8=BE*CE*EO*ER+MU*BH*CH
      T9=T1+T2
      T10=T3+T4
      T11=T5+T6
      P=OMEGA*BETA*(ALG1*T9+0.5*ALG2*SI2*T10+D*T11*ALAG2+FCO
     1*T8*ALG2)+2.*D*ALPHA*G2*T7*(B2-K22)
      GO TO 9
C
C     TRIGONOMETRIC CASE, GAMMA 2 IS IMAGINARY
C
    7 G22=ABS(A2+B2-K22)
      G2=SQRT(G22)
      AG2=A2/G2+G2
      ALOG2=A2-G22
      G2D=G2*D
      G22D=2.*G2D
      TSI=SIN(G2D)
      TCO=COS(G2D)
      TSI2=SIN(G22D)
      TCO2=1.-COS(G22D)
C
C     INTERMEDIATE ALGAEBRAIC STEPS
C
      N1=KC4*(G3/(G2*ER)*KC4*TSI+TCO)
      N2=KC4*AB/(OE*G2)*KC5*TSI
      N3=AB/(OM*G1)*KC4*G3/(G2*ER) KC5*TSI
      N4=1./(OM*G1)*((-OM*G2-(AB**2)*KC6/(OE*G2))*TSI+OM*G3*
     1KC4*TCO)
      N5=N1*N4+N2*N3
      P1=N4/N5
      P2=N2/N5
      P3=N3/N5
      P4=N1/N5
      Q1=KC2SQ*((G3/G2)*KC4*P4-AB/(OM*G2)*P2*KC5)*TSI+KC2SQ*
     1P4*TCO+KC1SQ
      Q2=KC2SQ*((AB/(OM*G2)*KC5*P1+(G3/G2)*KC4*P3)*TSI+P3*TC
     1O)
      Q3=(AB*(G3/G2)*KC4*P4+OE*G2*P2-(AB**2)/(OM*G2)*KC5*P2)
     1*TSI-((OE/ER)*G3*KC4*P2-AB*P4-AB*P4*KC5)*TCO+AB
      Q4=((AB**2)/(OM*G2)*KC5*P1+AB*(G3/G2)*P3*KC4-OE*G2*P1)
     1*TSI+(AB*P3+(OE/ER)*G3*KC4*P1+AB*KC5*P3)*TCO+(OE/ER)*G
     11
      Q5=Q2*Q3-Q1*Q4
      AE=Q1/Q5*CJ
      AH=Q2/Q5*CJ
      CE=P1*AE+P2*AH
      CH=P3*AE-P4*AH
      BE=1./(OE*G2)*((OE/ER)*G3*KC4*CE+AB*KC5*CH)
```

```
      BH=1./(OM*G2)*(AB*KC5*CE+OM*G3*KC4*CH)
      DE=KC4*CE
      DH=KC4*CH
      AE2=AE**2
      AH2=AH**2
      BE2=BE**2
      BH2=BH**2
      CE2=CE**2
      CH2=CH**2
      DE2=DE**2
      DH2=DH**2
      T1=(AE2+DE2)*EO
      T2=(AH2+DH2)*MU
      T3=(BE2+CE2)*EO*ER
      T4=(BH2+CH2)*MU
      T5=(CE2-BE2)*EO*ER
      T6=(CH2-BH2)*MU
      T7=BE*CH-BH*CE
      T8=BE*CE*EO*ER+MU*BH*CH
      T9=T1+T2
      T10=T3+T4
      T11=T5+T6
      P=OMEGA*BETA*(ALG1*T9+0.5*AG2*T10*TSI2+D*ALCG2*T11)-2.
     10*D*ALPHA*G2*T7*(B2+K22)
    9 M2=ABS(P/FPI)
      M3=M3+M2*A
      ALFA=ALFA+A
      IF (ALFA.GT.B) GO TO 2
      GO TO 1
    2 M4=M3/M1
      WRITE (6,19) ER,SCW,DRATIO,FRE,M4
   19 FORMAT (' ',3X,'FOR ER = ',F5.2,2X,'S/W = ',F5.2,2X,'D
     1/L = ',F6.4,2X,'FREQ = ',F5.2,1X,'GHZ',2X,'Z IS ',E14.
     17,1X,'OHMS',//)
   13 CONTINUE
   11 CONTINUE
   23 STOP
      END
```

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A general analysis of coplanar lines on a dielectric substrate is presented. The field equations are written in terms of Hertzian potentials and boundary conditions are applied to generate a system of linear equations. These equations are then Fourier transformed and reduced to a form where a computationally efficient solution can be obtained by the method of moments. Any desired accuracy may be obtained by expanding the transform of the current if the problem is one involving strip conductors or the electric

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field if there are slots. A one term expansion produces very good results. Both wavelength and characteristic impedance of the transmission line structure are obtained. Theoretical and experimental results are presented for coplanar strips.

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